نسخة أولية

غلاف الحقيبة

يتم إدراجه لاحقاً من قبل الإدارة العامة للمناهج

مقدمة

الحمد لله الذي علَّم بالقلم، علَّم الإنسان ما لم يعلم، والصلاة والسلام على من بُعث مُعلماً للناس وهادياً وبشيراً، وداعياً إلى الله بإذنه وسراجاً منيراً؛ فأخرج الناس من ظلمات الجهل والغواية، إلى نور العلم والهداية، نبينا ومعلمنا وقدوتنا الأول محمد بن عبدالله وعلى آله وصحبه أجمعين، أما بعد:

تسعى المؤسسة العامة للتدريب التقني والمهني لتأهيل الكوادر الوطنية المدربة القادرة على شغل الوظائف التقنية والفنية والمهنية المتوفرة في سوق العمل السعودي، ويأتي هذا الاهتمام نتيجة للتوجهات السديدة من لدن قادة هذا الوطن التي تصب في مجملها نحو إيجاد وطن متكامل يعتمد ذاتياً على الله ثم على موارده وعلى قوة شبابه المسلح بالعلم والإيمان من أجل الاستمرار قدماً في دفع عجلة التقدم التنموي، لتصل بعون الله تعالى لمصاف الدول المتقدمة صناعياً.

وقد خطت الإدارة العامة للمناهج خطوة إيجابية تتفق مع التجارب الدولية المتقدمة في بناء البرامج التدريبية، وفق أساليب علمية حديثة تحاكي متطلبات سوق العمل بكافة تخصصاته لتلبي تلك المتطلبات، وقد تمثلت هذه الخطوة في مشروع إعداد المعايير المهنية الوطنية ومن بعده مشروع المؤهلات المهنية الوطنية، والذي يمثل كل منهما في زمنه، الركيزة الأساسية في بناء البرامج التدريبية، إذ تعتمد المعايير وكذلك المؤهلات لاحقاً في بنائها على تشكيل لجان تخصصية تمثل سوق العمل والمؤسسة العامة للتدريب التقني والمهني بحيث تتوافق الرؤية العلمية مع الواقع العملي الذي تفرضه متطلبات سوق العمل، لتخرج هذه اللجان في النهاية بنظرة متكاملة لبرنامج تدريبي أكثر التصاقاً بسوق العمل، وأكثر واقعية في تحقيق متطلباته الأساسية.

وتتناول هذه الحقيبة التدريبية **" الاقتصاد الهندسي"** لمتدربي برنامج **" البكالوريوس "** في الكليات التقنية، موضوعات حيوية تتناول كيفية اكتساب المهارات اللازمة لهذا البرنامج لتكون مهاراتها رافداً لهم في حياتهم العملية بعد تخرجهم من هذا البرنامج. والإدارة العامة للمناهج وهي تضع بين يديك هذه الحقيبة التدريبية تأمل من الله عز وجلَّ أن تسهم بشكل مباشر في تأصيل المهارات الضرورية اللازمة، بأسلوب مبسط خالٍ من التعقيد.

والله نسأل أن يوفق القائمين على إعدادها والمستفيدين منها لما يحبه ويرضاه؛ إنه سميع مجيب الدعاء.

الإدارة العامة للمناهج

Contents

Preface

Course Description:

This course aims at providing the student with basic concepts of engineering economic analysis and its role in engineering decision making. It is designed to offer the students the tools needed for rigorous presentation of the effect of the time value of money on engineering problem solving and the capacity to act with efficient professionalism. The course introduced include foundations of engineering economy, nominal and effective interest rates, engineering economy factors, present worth analysis, annual worth analysis, rate of return analysis, benefit/cost analysis and public-sector economics, breakeven and payback analysis, and depreciation.

General Objectives:

The objective of this course is to give the working engineer an overview of the economics methods employed in effective engineering decisions as related to the designing, planning and implementation of successful projects

Estimated time for this course: 32 Training hours

Chapter 1	**Fundamentals of Engineering Economy**	5 Training hours
Chapter 2	**Factors: How Time and Interest Affect Money**	5 Training hours
Chapter 3	**Nominal and Effective Interest Rates**	3 Training hours
Chapter 4	**Present Worth Analysis**	3 Training hours
Chapter 5	**Annual Worth Analysis**	3 Training hours
Chapter 6	**Rate of Return (ROR) Analysis**	4 Training hours
Chapter 7	**Benefit/Cost Analysis and Public-Sector Economics**	3 Training hours
Chapter 8	**Breakeven and Payback Analysis**	3 Training hours
Chapter 9	**Depreciation Methods**	3 Training hours

Detailed Objectives:

Trainer will be able to:

1. Recognize the time value of money and the factors that allow the conversion of money through time.
2. Identify and compare different interest rates i.e., simple, compound, MARR, ROR, nominal and effective.
3. Convert given cash-based problems into a cash flow using a cash flow diagram.
4. Compute equivalent values for time-based cash flows of varying complexities.

5. Compare projects alternatives by different techniques based on equivalent Present Worth (PW), Future Worth (FW), Capitalized Cost (CC), Payback Period (P**b**P), Annual worth (AW) values and Benefit Cost ratios (B/C).
6. Compute depreciations related to projects using Straight Line (SL) and Declining Balance (DB).
7. Use EXCEL spreadsheets and financial functions to model and solve engineering economic analysis problems.

Chapter 1

Fundamentals of Engineering Economy

Chapter 1
Fundamentals of Engineering Economy

General Objective:

Trainee will be able to understand the basic concepts and terminology necessary for engineering economy

Estimated time for this chapter: 5 Training hours

Detailed Objectives:

1. Engineering economics: description and role in decision making process.
2. Performing an Engineering Economy Study.
3. Interest Rate and Rate of Return (ROR).
4. Economic Equivalence.
5. Terminology and Symbols.
6. Simple and Compound Interest.
7. Cash Flows: Their Estimation and Diagramming (CFD).
8. Minimum Attractive Rate of Return (MARR).
9. Spreadsheets use in engineering economy.

Introduction

The need for engineering economy is primarily motivated by the work that engineers do in performing analyses, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of ***making decisions.*** These decisions involve the fundamental elements of ***cash flows of money, time, and interest rates.*** This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

1.1. Engineering economics: description and role in decision making process

Decisions are made routinely to choose one alternative over another by individuals in everyday life; by engineers on the job; by managers who supervise the activities of others; by corporate presidents who operate a business; and by government officials who work for the public good. Most decisions involve money, called capital or capital funds, which is usually limited in amount. The decision of where and how to invest this limited capital is motivated by a primary goal of adding value as future, anticipated results of the selected alternative are realized. Engineers play a vital role in capital investment decisions based upon their ability and experience to design, analyze, and synthesize. The factors upon which a decision is based are commonly a combination of economic and noneconomic elements. Engineering economy deals with the economic factors. By definition,

Engineering economy involves formulating, estimating, and evaluating the expected economic outcomes of alternatives designed to accomplish a defined purpose. Mathematical techniques simplify the economic evaluation of alternatives.

Other terms that mean the same as engineering economy are engineering economic analysis, capital allocation study, economic analysis, and similar descriptors.

People make decisions; computers, mathematics, concepts, and guidelines assist people in their decision-making process. Since most decisions affect what will be done, the time frame of engineering economy is primarily the future. Therefore, the numbers used in engineering economy are ***best estimates of what is expected to occur***. The estimates and the decision usually involve four essential elements:

- Cash flows
- Times of occurrence of cash flows
- Interest rates for time value of money
- Measure of economic worth for selecting an alternative

Since the estimates of cash flow amounts and timing are about the future, they will be somewhat different than what is actually observed, due to changing circumstances and unplanned events. In short, the variation between an amount or time estimated now and that observed in the future is caused by the stochastic (random) nature of all economic events. ***Sensitivity analysis*** is utilized to determine how a decision might change according to varying estimates, especially those expected to vary widely. Example (1-1) illustrates the fundamental nature of variation in estimates and how this variation may be included in the analysis at a very basic level.

Example (1-1)

An engineer is performing an analysis of warranty costs for drive train repairs within the first year of ownership of luxury cars purchased in the United States. He found the average cost (to the nearest dollar) to be $570 per repair from data taken over a 5-year period.

Year	2006	2007	2008	2009	2010
Average Cost, $/repair	525	430	619	650	625

What range of repair costs should the engineer use to ensure that the analysis is sensitive to changing warranty costs?

Solution

At first glance the range should be approximately –25% to +15% of the $570 average cost to include the low of $430 and high of $650. However, the last 3 years of costs are higher and more consistent with an average of $631. The observed values are approximately ±3% of this more recent average.

- If the analysis is to use the most recent data and trends, a range of, say, ±5% of $630 is recommended.
- If, however, the analysis is to be more inclusive of historical data and trends, a range of, say, ±20% or ±25% of $570 is recommended.

The criterion used to select an alternative in engineering economy for a specific set of estimates is called a measure of worth. The measures developed and used in this text are:

- Present worth (PW)
- Future worth (FW)
- Annual worth (AW)
- Rate of return (ROR)
- Benefit/cost (B/C)
- Capitalized cost (CC)
- Payback period (PP)
- Economic value added (EVA)
- Cost Effectiveness (CE)

All these measures of worth account for the fact that money makes money over time. This is the concept of the time ***value of money***.

It is a well-known fact that money makes money. The time value of money explains the change in the amount of money over time for funds that are owned (invested) or owed (borrowed). This is the most important concept in engineering economy.

The time value of money is very obvious in the world of economics. If we decide to invest capital (money) in a project today, we inherently expect to have more money in the future than we invested. If we borrow money today, in one form or another, we expect to return the original amount plus some additional amount of money. Engineering economics is equally well suited for the future and for the analysis of past cash flows in order to determine if a specific criterion (measure of worth) was attained.

1.2. Performing an Engineering Economy Study

An engineering economy study involves many elements: problem identification, definition of the objective, cash flow estimation, financial analysis, and decision making.

Implementing a structured procedure is the best approach to select the best solution to the problem.

The steps in an engineering economy study are as follows:

1- Identify and understand the problem; identify the objective of the project.
2- Collect relevant, available data and define viable solution alternatives.
3- Make realistic cash flow estimates.
4- Identify an economic measure of worth criterion for decision making.
5- Evaluate each alternative; consider noneconomic factors; use sensitivity analysis as needed.
6- Select the best alternative.
7- Implement the solution and monitor the results.

Technically, the last step is not part of the economy study, but it is, of course, a step needed to meet the project objective. There may be occasions when the best economic alternative

requires more capital funds than are available, or significant noneconomic factors preclude the most economic alternative from being chosen. Accordingly, steps 5 and 6 may result in selection of an alternative different from the economically best one. Also, sometimes more than one project may be selected and implemented. This occurs when projects are independent of one another. In this case, steps 5 through 7 vary from those above. Figure (1-1) illustrates the steps above for one alternative. Descriptions of several of the elements in the steps are important to understand.

Problem Description and Objective Statement: A succinct statement of the problem and primary objective(s) is very important to the formation of an alternative solution. As an illustration, assume the problem is that a coal-fueled power plant must be shut down by 2030 due to the production of excessive sulfur dioxide. The objectives may be to generate the forecasted electricity needed for 2030 and beyond, plus to not exceed all the projected emission allowances in these future years.

Alternatives: These are stand-alone descriptions of viable solutions to problems that can meet the objectives. Words, pictures, graphs, equipment and service descriptions, simulations, etc. define each alternative. The best estimates for parameters are also part of the alternative. Some parameters include equipment first cost, expected life, salvage value (estimated trade-in, resale, or market value), and annual operating cost (AOC), which can also be termed maintenance and operating (M&O) cost, and subcontract cost for specific services. If changes in income (revenue) may occur, this parameter must be estimated.

Cash Flows: All cash flows are estimated for each alternative. Since these are future expenditures and revenues, the results of step 3 usually prove to be inaccurate when an alternative is actually in place and operating. When cash flow estimates for specific parameters are expected to vary significantly from a point estimate made now, risk and sensitivity analyses (step 5) are needed to improve the chances of selecting the best alternative. Sizable variation is usually expected in estimates of revenues, AOC, salvage values, and subcontractor costs.

Engineering Economy Analysis: The techniques and computations that you will learn and use throughout this text utilize the cash flow estimates, time value of money, and a selected measure of worth. The result of the analysis will be one or more numerical values; this can be in one of several terms, such as money, an interest rate, number of years, or a probability. In the end, a selected measure of worth mentioned in the previous section will be used to select the best alternative.

Before an economic analysis technique is applied to the cash flows, some decisions about what to include in the analysis must be made. Two important possibilities are taxes and inflation. Federal, state or provincial, county, and city taxes will impact the costs of every alternative. An after-tax analysis includes some additional estimates and methods compared to a before-tax analysis. If taxes and inflation are expected to impact all alternatives equally, they may be disregarded in the analysis. However, if the size of these projected costs is important, taxes and inflation should be considered. Also, if the impact of inflation over time is important to the decision, an additional

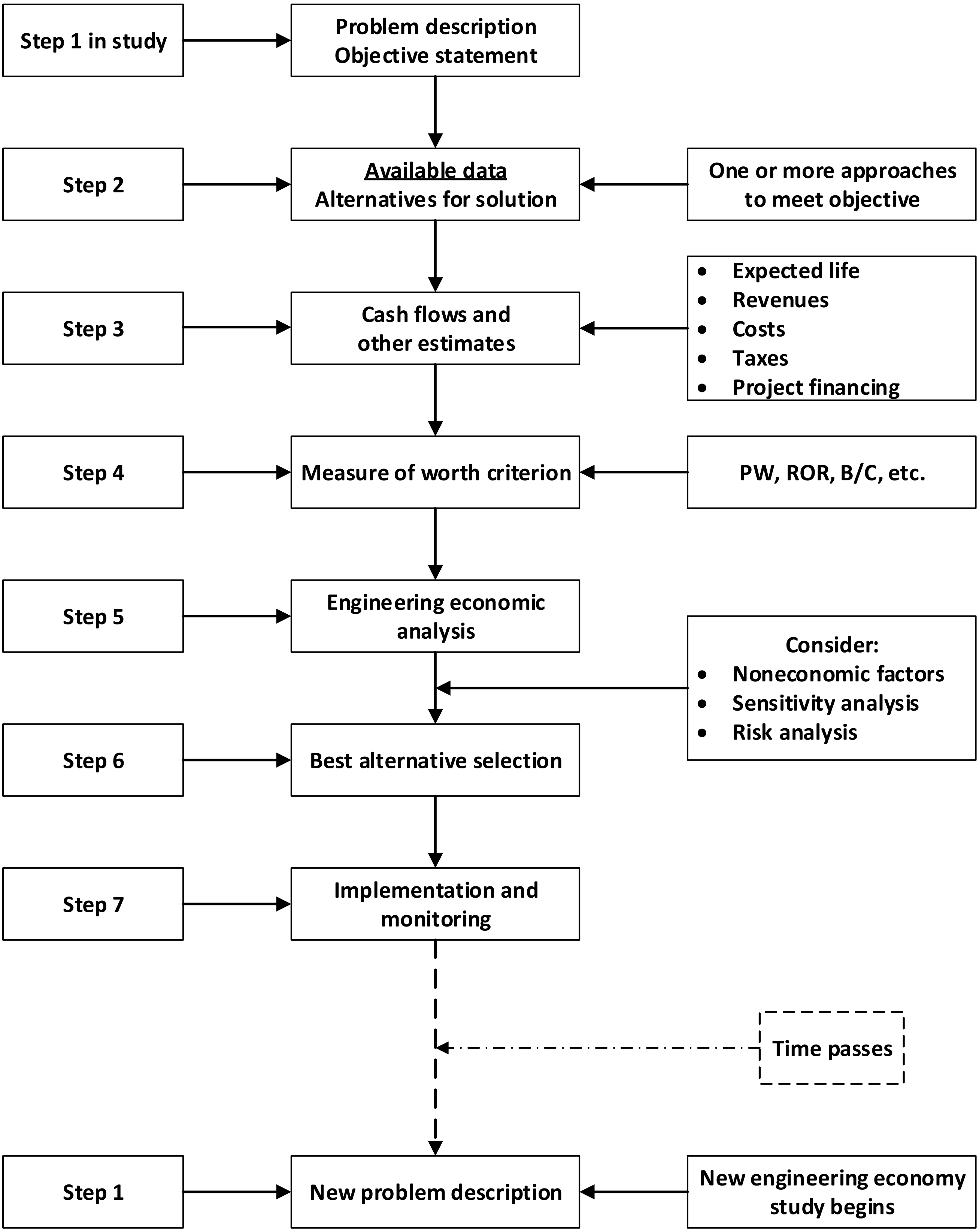


Figure (1-1): Steps in an engineering economy study

Selection of the Best Alternative: The measure of worth is a primary basis for selecting the best economic alternative. For example, if alternative A has a rate of return (ROR) of 15.2% per year and alternative B will result in an ROR of 16.9% per year, B is better economically. However, there can always be ***noneconomic*** or ***intangible factors*** that must be considered and that may alter the decision. There are many possible noneconomic factors; some typical ones are:

- Market pressures, such as need for an increased international presence
- Availability of certain resources, e.g., skilled labor force, water, power, tax incentives
- Government laws that dictate safety, environmental, legal, or other aspects
- Corporate management's or the board of director's interest in a particular alternative
- Goodwill offered by an alternative toward a group: employees, union, county, etc.

As indicated in Figure (1-1), once all the economic, noneconomic, and risk factors have been evaluated, a final decision of the "best" alternative is made. At times, only one viable alternative is identified. In this case, the do-nothing (DN) alternative may be chosen provided the measure of worth and other factors result in the alternative being a poor choice. The do-nothing alternative maintains the status quo.

1.3. Interest Rate and Rate of Return (ROR)

Interest is the manifestation of the time value of money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an amount of interest—interest ***paid*** and interest earned. These are illustrated in Figure (1–2). Interest is paid when a person or organization borrowed money (obtained a loan) and repays a larger amount over time. Interest is ***earned*** when a person or organization saved, invested, or lent money and obtains a return of a larger amount over time. The numerical values and formulas used are the same for both perspectives, but the interpretations are different.

Interest paid on borrowed funds (a loan) is determined using the original amount, also called the principal,

$$Interest = amount\ owed\ now - principal \qquad (1-1)$$

When interest paid over a specific time unit is expressed as a percentage of the principal, the result is called the ***interest rate***.

$$Interest\ rate\ (\%) = \frac{interest\ accrued\ per\ time\ unit}{principal} \times 100\% \qquad (1-2)$$

The time unit of the rate is called the ***interest period***. By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as 1% per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, 8.5%, a 1-year interest period is assumed.

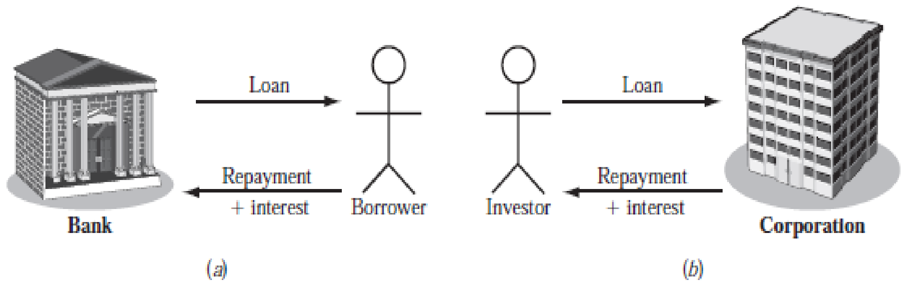


Figure (1-2): (a) Interest paid over time to lender. (b) Interest earned over time by investor.

Example (1-2)

An employee at LaserKinetics.com borrows $10,000 on May 1 and must repay a total of $10,700 exactly 1 year later. Determine the interest amount and the interest rate paid.

Solution

The perspective here is that of the borrower since $10,700 repays a loan. Apply Equation (1-1) to determine the interest paid.

$$Interest\ paid = \$10{,}700 - 10{,}000 = \$700$$

Equation (1-2) determines the interest rate paid for 1 year.

$$Percent\ interest\ rate\ (\%) = \frac{\$700}{\$10{,}000} \times 100\% = 7\%\ per\ year$$

Example (1-3)

Stereophonics, Inc., plans to borrow $20,000 from a bank for 1 year at 9% interest for new recording equipment.

1- Compute the interest and the total amount due after 1 year.
2- Construct a column graph that shows the original loan amount and total amount due after 1 year used to compute the loan interest rate of 9% per year.

Solution

1- Compute the total interest accrued by solving Equation (1-2) for interest accrued.

$$Interest = \$20{,}000 \times 0.09 = \$1800$$

The total amount due is the sum of principal and interest.

$$Total\ due = \$20{,}000 + 1800 = \$21{,}800$$

2- Figure (1-3) shows the values used in Equation (1-2): $1800 interest, $20,000 original loan principal, 1-year interest period.

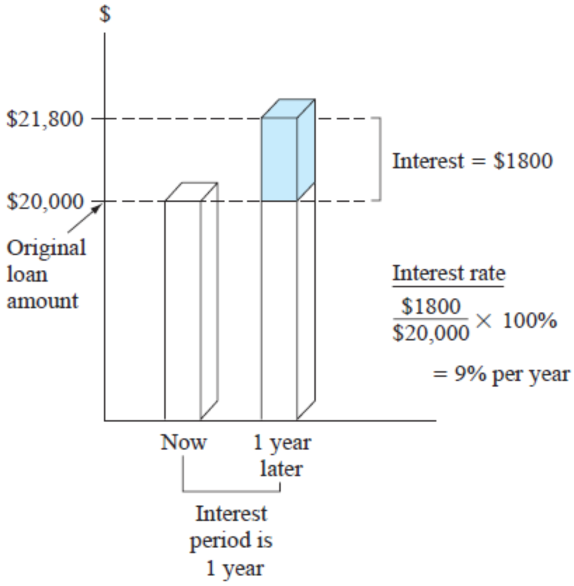


Figure (1-3): Values used to compute an interest rate of 9% per year.

Example (1-3)

From the perspective of a saver, a lender, or an investor, ***interest earned*** (Figure (1–2b)) is the final amount minus the initial amount, or principal.

$$interest\ earned\ =\ total\ amount\ now -\ principal \qquad (1-3)$$

Interest earned over a specific period of time is expressed as a percentage of the original amount and is called ***rate of return (ROR).***

$$Rate\ of\ return\ (\%) = \frac{interest\ accrued\ per\ time\ unit}{principal} \times 100\% \quad (1-4)$$

The time unit for rate of return is called the interest period, just as for the borrower's perspective. Again, the most common period is 1 year. The term return on investment (ROI) is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs. The numerical values in Equations (1-2) and (1-4) are the same, but the term interest rate paid is more appropriate for the borrower's perspective, while the rate of return earned is better for the investor's perspective.

Example (1-4)

1- Calculate the amount deposited 1 year ago to have $1000 now at an interest rate of 5% per year.
2- Calculate the amount of interest earned during this time period.

Solution

1- The total amount accrued ($1000) is the sum of the original deposit and the earned interest. If X is the original deposit,

$$Total\ accrued = deposit + deposit(interest\ rate)$$
$$\$1000 = X + X(0.05) = X(1 + 0.005) = 1.05X$$

The original deposit is

$$X = \frac{1000}{1.05} = \$952.38$$

2- Apply Equation (1-3) to determine the interest earned.

$$Interest = \$1000 - 952.38 = \$47.62$$

In Examples (1-2) to (1-4) the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved, e.g., the amount of interest after 3 years, it is necessary to state whether the interest is accrued on a simple or compound basis from one period to the next. This topic is covered later in this chapter.

Since ***inflation*** can significantly increase an interest rate, some comments about the fundamentals of inflation are warranted at this early stage. Inflation represents a decrease in the value of a given currency. That is, \$10 now will not purchase the same amount of gasoline for your car (or most other things) as \$10 did 10 years ago. The changing value of the currency affects market interest rates.

1.4. Terminology and Symbols

The equations and procedures of engineering economy utilize the following terms and symbols. Sample units are indicated.

symbols	***Definitions***
P	Value or amount of money at a time designated as the present or time 0. Also, P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); monetary units, such as dollars
F	Value or amount of money at some future time. Also, F is called future worth (FW) and future value (FV); dollars
A	Series of consecutive, equal, end-of-period amounts of money. Also, A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, euros per month
n	Number of interest periods; years, months, day
i	Interest rate per time period; percent per year, percent per month
t	Time, stated in periods; years, months, days

Example (1-5)

Today, Julie borrowed \$5000 to purchase furniture for her new house. She can repay the loan in either of the two ways described below. Determine the engineering economy symbols and their value for each option.

1- Five equal annual installments with interest based on 5% per year.
2- One payment 3 years from now with interest based on 7% per year.

Solution

1- The repayment schedule requires an equivalent annual amount A, which is unknown.

$$P = \$5000 \quad i = 5\% \text{ per year} \quad n = 5 \text{ years} \quad A = ?$$

2- Repayment requires a single future amount F, which is unknown.

$$P = \$5000 \quad i = 7\% \text{ per year} \quad n = 3 \text{ years} \quad F = ?$$

Example (1-6)

You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

Solution

All five symbols are present, but the future value in year 6 is the unknown.

P = \$5000
A = \$1000 per year for 5 years
F = ? at end of year 6
i = 6% per year
n = 5 years for the A series and 6 for the F value

Example (1-7)

Last year Jane's grandmother offered to put enough money into a savings account to generate \$5000 in interest this year to help pay Jane's expenses at college.

1- Identify the symbols,
2- Calculate the amount that had to be deposited exactly 1 year ago to earn \$5000 in interest now, if the rate of return is 6% per year.

Solution

1- Symbols P (last year is -1) and F (this year) are needed.

P = ?
i = 6% per year
n = 1 year
$F = P$ + interest = ? + \$5000

2- Let F = total amount now and P = original amount. We know that $F - P = \$5000$ is accrued interest. Now we can determine P. Refer to Equations (1-1) through (1- 4).

$$F = P + Pi$$

The \$5000 interest can be expressed as

$$Interest = F - P = P + Pi - P = Pi$$

$$\$5000 = P(0.06); \quad P = \frac{\$5000}{0.06} = \$83{,}333.33$$

1.5. Cash Flows: Estimation and Diagramming (CFD)

As mentioned in earlier sections, cash flows are the amounts of money estimated for future projects or observed for project events that have taken place. All cash flows occur during specific time periods, such as 1 month, every 6 months, or 1 year. Annual is the most common time period. For example, a payment of \$10,000 once every year in December for 5 years is a series of 5 outgoing cash flows. And an estimated receipt of \$500 every month for 2 years is a series of 24 incoming cash flows. Engineering economy bases its computations on the timing, size, and direction of cash flows.

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A plus sign indicates a cash inflow.

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business activity. A negative or minus sign indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

Of all the steps in Figure (1–1) that outline the engineering economy study, estimating cash flows (step 3) is the most difficult, primarily because it is an attempt to predict the future. Some examples of cash flow estimates are shown here. As you scan these, consider how the cash inflow or outflow may be estimated most accurately.

Cash Inflow Estimates:

Income: -$150,000 per year from sales of solar-powered watches
Savings: -$24,500 tax savings from capital loss on equipment salvage
Receipt: -$750,000 received on large business loan plus accrued interest
Savings: -$150,000 per year saved by installing more efficient air conditioning
Revenue: -$50,000 to -$75,000 per month in sales for extended battery life iPhones

Cash Outflow Estimates:

Operating costs: -$230,000 per year annual operating costs for software services
First cost: -$800,000 next year to purchase replacement earthmoving equipment
Expense: -$20,000 per year for loan interest payment to bank
Initial cost: -$1 to -$1.2 million in capital expenditures for a water recycling unit

All of these are ***point estimates*****,** that is, ***single-value estimates*** for cash flow elements of an alternative, except for the last revenue and cost estimates listed above. They provide a ***range estimate*****,** because the persons estimating the revenue and cost do not have enough knowledge or experience with the systems to be more accurate. Once all cash inflows and outflows are estimated (or determined for a completed project), the ***net cash flow*** for each time period is calculated.

$$Net\ cash\ flow\ = cash\ inflows - \ cash\ outflows \qquad (1-5)$$

$$NCF\ = R - D \qquad (1-6)$$

where NCF is net cash flow, R is receipts, and D is disbursements.

At the beginning of this section, the ***timing, size, and direction of cash flows*** were mentioned as important. Because cash flows may take place at any time during an interest period, as a matter of convention, all cash flows are assumed to occur at the end of an interest period.

The end-of-period convention means that all cash inflows and all cash outflows are assumed to take place at the end of the interest period in which they actually occur. When several inflows and outflows occur within the same period, the net cash flow is assumed to occur at the end of the period.

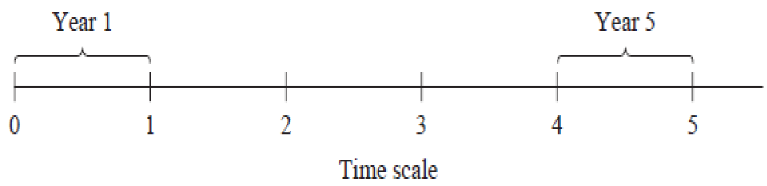


Figure (1-4): A typical cash flow time scale for 5 years.

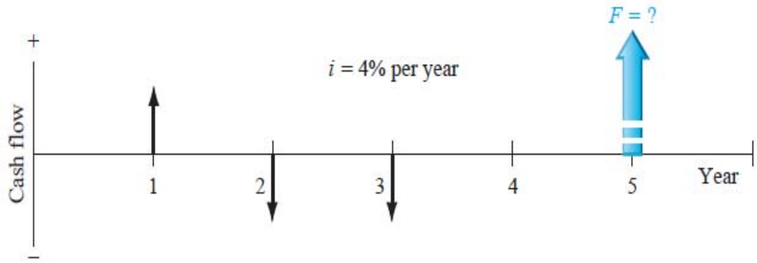


Figure (1-5): Example of positive and negative cash flows.

It is important to understand that future (*F*) and uniform annual (*A*) amounts are located at the end of the interest period, which is not necessarily December 31. Remember, end of the period means end of interest period, not end of calendar year.

The ***cash flow diagram*** is a very important tool in an economic analysis, especially when the cash flow series is complex. It is a graphical representation of cash flows drawn on the *y* axis with a time scale on the *x* axis. The diagram includes what is known, what is estimated, and what is needed. That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

Cash flow diagram time $t = 0$ is the present, and $t = 1$ is the end of time period 1. We assume that the periods are in years for now. The time scale of Figure (1–4) is set up for 5 years. Since the end-of-year convention places cash flows at the ends of years, the "1" marks the end of year 1.

The direction of the arrows on the diagram is important to differentiate income from outgo.

- A vertical arrow pointing up indicates a positive cash flow.
- A down-pointing arrow indicates a negative cash flow.
- We will use a bold, colored arrow to indicate what is unknown and to be determined.

For example, if a future value *F* is to be determined in year 5, a wide, colored arrow with *F*= ? is shown in year 5. The interest rate is also indicated on the diagram. Figure (1–5) illustrates a cash inflow at the end of year 1, equal cash outflows at the end of years 2 and 3, an interest rate of 4% per year, and the unknown future value *F* after 5 years. The arrow for the unknown value is generally drawn in the opposite direction from the other cash flows; however, the engineering economy computations will determine the actual sign on the *F* value.

Before the diagramming of cash flows, a perspective or vantage point must be determined so that + or – signs can be assigned and the economic analysis performed correctly. Assume you borrow \$8500 from a bank today to purchase an \$8000 used car for cash next week, and you plan to spend the remaining \$500 on a new paint job for the car two weeks from now.

There are several perspectives possible when developing the cash flow diagram—those of the borrower (that's you), the banker, the car dealer, or the paint shop owner. The cash flow signs and amounts for these perspectives are as follows.

Perspective	*Activity*	*Cash flow with Sign, $*	*Time, week*
You	Borrow	+8500	0
	Buy car	-8000	1
	Paint job	-500	2
Banker	Lender	-8500	0
Car dealer	Car sale	+8000	1
Painter	Paint job	+500	2

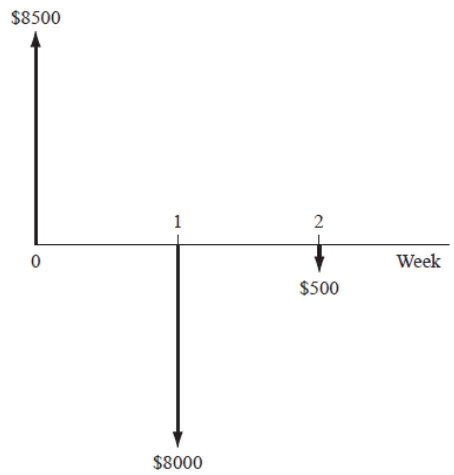


Figure (1-6): Cash flows from perspective of borrower for loan and purchases.

One, and only one, of the perspectives is selected to develop the diagram. For your perspective, all three cash flows are involved and the diagram appears as shown in Figure (1–6) with a time scale of weeks. Applying the end-of-period convention, you have a receipt of +$8500 now (time 0) and cash outflows of -$8000 at the end of week 1, followed by -$500 at the end of week 2.

Example (1-8)

An electrical engineer wants to deposit an amount P now such that she can withdraw an equal annual amount of $A1$ = $2000 per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of $A2$ = $3000 per year for the following 3 years. How would the cash flow diagram appear if i= 8.5% per year?

Solution

The cash flows are shown in Figure (1–7). The negative cash outflow P occurs now. The withdrawals (positive cash inflow) for the $A1$ series occur at the end of years 1 through 5, and $A2$ occurs in years 6 through 8.

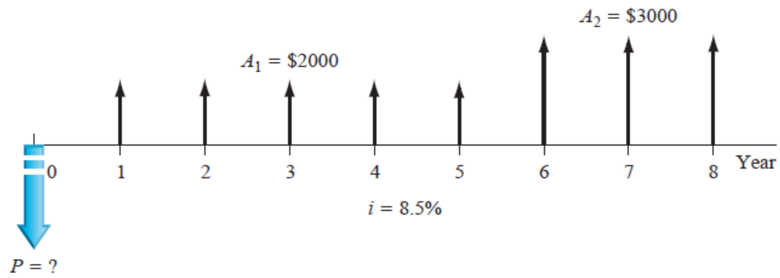


Figure (1-7): Cash flow diagram with two different A series, Example (1-8).

1.6. Economic Equivalence

Economic equivalence is a fundamental concept upon which engineering economy computations are based. Before we delve into the economic aspects, think of the many types of equivalency we may utilize daily by transferring from one scale to another. Some example transfers between scales are as follows:

Length:

12 inches = 1 foot 3 feet = 1 yard 39.370 inches = 1 meter
100 centimeters = 1 meter 1000 meters = 1 kilometer 1 kilometer = 0.621 mile

Pressure:

1 atmosphere =1 newton/meter ^{2} = 10 ^{3} pascal = 1 kilopascal

Speed:

1 mile = 1.609 kilometers 1 hour = 60 minutes
110 kilometers per hour (kph) = 68.365 miles per hour (mph)
68.365 mph = 1.139 miles per minute

Economic equivalence is a combination of interest rate and time value of money to determine the different amounts of money at different points in time that are equal in economic value.

As an illustration, if the interest rate is 6% per year, \$100 today (present time) is equivalent to \$106 one year from today.

$$Amount\ accrued = 100 + 100(0.06) = 100(1 + 0.06) = \$106$$

If someone offered you a gift of \$100 today or \$106 one year from today, it would make no difference which offer you accepted from an economic perspective. In either case you have \$106 one year from today. However, the two sums of money are equivalent to each other *only* when the interest rate is 6% per year. At a higher or lower interest rate, \$100 today is not equivalent to \$106 one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of \$100 now is equivalent to \$100/1.06 = \$94.34 one year ago at an interest rate of 6% per year. From these illustrations, we can state the following: \$94.34 last year, \$100 now, and \$106 one year from now are equivalent at an interest rate of 6% per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1-year interest periods.

$$\frac{\$6}{\$100} \times 100\% = 6\%\ per\ year\ \ and\ \ \frac{\$5.66}{\$94.34} \times 100\% = 6\%\ per\ year$$

The cash flow diagram in Figure (1–8) indicates the amount of interest needed each year to make these three different amounts equivalent at 6% per year.

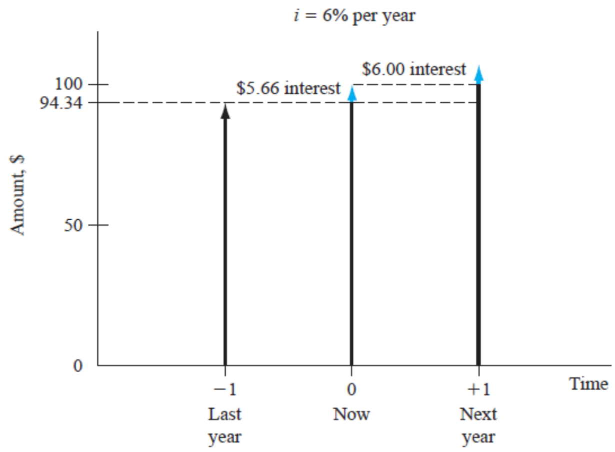


Figure (1-8): Equivalence of money at 6% per year interest.

Example (1-9)

Manufacturers make backup batteries for computer systems available to Batteries+ dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Batteries+ outlet. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.

1) The amount of $98 now is equivalent to a cost of $105.60 one year from now.
2) A truck battery cost of $200 one year ago is equivalent to $205 now.
3) A $38 cost now is equivalent to $39.90 one year from now.
4) A $3000 cost now is equivalent to $2887.14 one year earlier.
5) The carrying charge accumulated in 1 year on an investment of $20,000 worth of batteries is $1000.

Solution

1) Total amount accrued = 98(1.05) = $102.90 ≠ $105.60; therefore, it is false. Another way to solve this is as follows: Required original cost is 105.60/1.05 = $100.57 ≠ $98.
2) Equivalent cost 1 year ago is 205.00/1.05 = $195.24 ≠ $200; therefore, it is false.
3) The cost 1 year from now is $38(1.05) = $39.90; true.
4) Cost now is 2887.14(1.05) = $3031.50 ≠ $3000; false.
5) The charge is 5% per year interest, or $20,000(0.05) = $1000; true.

1.7. Simple and Compound Interest

Simple interest:

Is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as:

$$Simple\ interest = (principal)(number\ of\ periods)(interest\ rate) \qquad (1-7)$$

$$I = Pni$$

where I is the amount of interest earned or paid and the interest rate i is expressed in decimal form.

Example (1-10)

GreenTree Financing lent an engineering company \$100,000 to retrofit an environmentally unfriendly building. The loan is for 3 years at 10% per year simple interest. How much money will the fi rm repay at the end of 3 years?

Solution

The interest for each of the 3 years is

Interest per year = \$100,000(0.10) = \$10,000

Total interest for 3 years from Equation (1-7) is

Total interest = \$100,000(3)(0.10) = \$30,000

The amount due after 3 years is

Total due = \$100,000 + 30,000 = \$130,000

The interest accrued in the first year and in the second year does not earn interest. The interest due each year is \$10,000 calculated only on the \$100,000 loan principal.

Compound interest:

In most financial and economic analyses, we use compound interest calculations. For compound interest*,* the interest accrued for each interest period is calculated on the principal plus the total amount of interest accumulated in all previous periods. Thus, compound interest means interest on top of interest.

Compound interest reflects the effect of the time value of money on the interest also. Now the interest for one period is calculated as:

$$Compound\ interest = (principal = all\ accrued\ interest)(interest\ rate) \quad (1-8)$$

In mathematical terms, the interest I_t for time period t may be calculated using the relation.

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j\right)(i) \qquad (1-9)$$

1.8. Minimum Attractive Rate of Return

Engineering alternatives are evaluated upon the prognosis that a reasonable ROR can be expected. Therefore, some reasonable rate must be established for the selection criteria (step 4) of the engineering economy study (Figure(1–1)).

The Minimum Attractive Rate of Return (MARR) is a reasonable rate of return established for the evaluation and selection of alternatives. A project is not economically viable unless it is expected to return at least the MARR. MARR is also referred to as the hurdle rate, cutoff rate, benchmark rate, and minimum acceptable rate of return.

Figure (1–9) indicates the relations between different rate of return values. In the United States, the current U.S. Treasury Bill return is sometimes used as the benchmark safe rate. The MARR will always be higher than this, or a similar, safe rate. The MARR is not a rate that is calculated as a ROR. The MARR is established by (financial) managers and is used as a criterion against which an alternative's ROR is measured, when making the accept/reject investment decision.

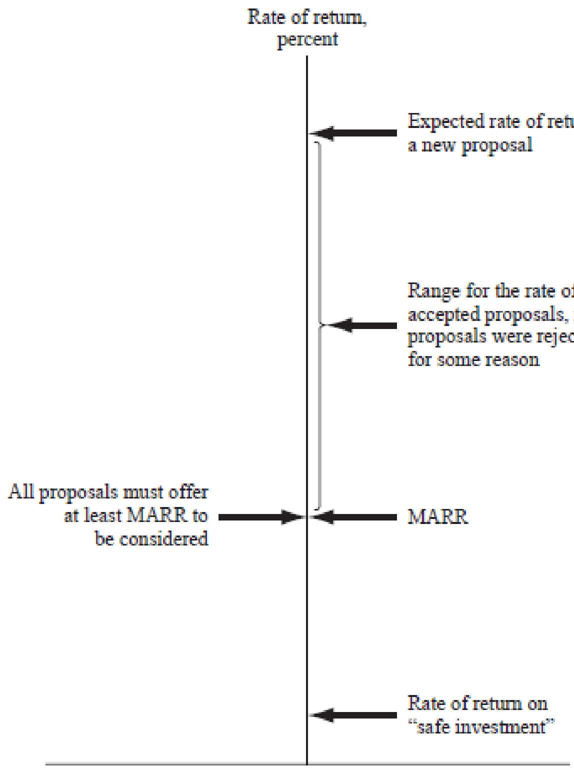


Figure (1-9): Size of MAAR relative to other rate of return values.

In general, capital is developed in two ways—equity financing and debt financing. A combination of these two is very common for most projects.

Equity financing: The corporation uses its own funds from cash on hand, stock sales, or retained earnings. Individuals can use their own cash, savings, or investments. In the example above, using money from the 5% savings account is equity financing.

Debt financing: The corporation borrows from outside sources and repays the principal and interest according to some schedule, much like the plans in Table (1–1). Sources of debt capital may be bonds, loans, mortgages, venture capital pools, and many others. Individuals, too, can utilize debt sources, such as the credit card (15% rate) and bank options (9% rate) described above.

Combinations of debt-equity financing mean that a ***weighted average cost of capital (WACC)*** results. If the HDTV is purchased with 40% credit card money at 15% per year and 60% savings account funds earning 5% per year, the weighted average cost of capital is

$$0.4(15) + 0.6(5) = 9\% \text{ per year.}$$

For a corporation, the established MARR used as a criterion to accept or reject an investment alternative will usually be equal to or higher than the WACC that the corporation must bear to obtain the necessary capital funds. So, the inequality

$$ROR \geq MARR > WACC \qquad (1-11)$$

must be correct for an accepted project. Exceptions may be government-regulated requirements (safety, security, environmental, legal, etc.), economically lucrative ventures expected to lead to other opportunities, etc.

Often there are many alternatives that are expected to yield a ROR that exceeds the MARR as indicated in Figure (1–9), but there may not be sufficient capital available for all, or the project's risk may be estimated as too high to take the investment chance. Therefore, new projects that are undertaken usually have an expected return at least as great as the return on another alternative that is not funded. The expected rate of return on the unfunded project is called the ***opportunity cost.***

The opportunity cost is the rate of return of a forgone opportunity caused by the inability to pursue a project. Numerically, it is the largest rate of return of all the projects not accepted (forgone) due to the lack of capital funds or other resources. When no specific MARR is established, the de facto MARR is the opportunity cost, i.e., the ROR of the first project not undertaken due to unavailability of capital funds.

1.9. Spreadsheets use in engineering economy

The functions on a computer spreadsheet can greatly reduce the amount of hand work for equivalency computations involving *compound interest* and the terms P , F , A , i , and n . The use of a calculator to solve most simple problems is preferred by many students and professors. However, as cash flow series become more complex, the spreadsheet offers a good alternative. Microsoft Excel is used throughout this book because it is readily available and easy to use.

A total of seven Excel functions can perform most of the fundamental engineering economy calculations. The functions are great supplemental tools, but they do not replace the understanding of engineering economy relations, assumptions, and techniques. Using the symbols P , F , A , i , and n defined in the previous section, the functions most used in engineering economic analysis are formulated as follows.

- To find the present value *P:* = **PV(*i*%, *n*, *A*, *F*)**
- To find the future value *F:* = **FV(*i*%, *n*, *A*, *P*)**
- To find the equal, periodic value *A:* = **PMT(*i*%, *n*, *P*, *F*)**
- To find the number of periods *n:* = **NPER(*i*%, *A*, *P*, *F*)**
- To find the compound interest rate *i:* = **RATE(*n*, *A*, *P*, *F*)**
- To find the compound interest rate *i:* = **IRR(first_cell:last_cell)**
- To find the present value *P* of any series: = **NPV(i %, second_cell:last_cell)** + **first_cell**

If some of the parameters don't apply to a particular problem, they can be omitted and zero is assumed. For readability, spaces can be inserted between parameters within parentheses. If the parameter omitted is an interior one, the comma must be entered. The last two functions require that a series of numbers be entered into contiguous spreadsheet cells, but the first five can be used with no supporting data. In all cases, the function must be preceded by an equals sign (+) in the cell where the answer is to be displayed.

Example (1-11)

A Japan-based architectural fi rm has asked a United States–based software engineering group to infuse GPS sensing capability via satellite into monitoring software for high-rise structures in order to detect greater than expected horizontal movements. This software could be very beneficial as an advance warning of serious tremors in earthquake-prone areas in Japan and the United States. The inclusion of accurate GPS data is estimated to increase annual revenue over that for the current software system by $200,000 for each of the next 2 years, and by $300,000 for each of years 3 and 4. The planning horizon is only 4 years due to the rapid advances made internationally in building-monitoring software. Develop spreadsheets to answer the questions below.

1) Determine the total interest and total revenue after 4 years, using a compound rate of return of 8% per year.
2) Repeat part (1) if estimated revenue increases from $300,000 to $600,000 in years 3 and 4.
3) Repeat part (1) if inflation is estimated to be 4% per year. This will decrease the real rate of return from 8% to 3.85% per year.

Solution

Refer to Figure (1–10) *a* to *d* for the solutions. All the spreadsheets contain the same information, but some cell values are altered as required by the question. (Actually, all the questions can be answered on one spreadsheet by changing the numbers. Separate spreadsheets are shown here for explanation purposes only.)

The Excel functions are constructed with reference to the cells, not the values themselves, so that sensitivity analysis can be performed without function changes. This approach treats the value in a cell as a *global variable* for the spreadsheet. For example, the 8% rate in cell B2 will be referenced in all functions as B2, not 8%. Thus, a change in the rate requires only one alteration in the cell B2 entry, not in every relation where 8% is used.

1) Figure (1–10 a) shows the results, and Figure (1–10 b) presents all spreadsheet relations for estimated interest and revenue (yearly in columns C and E, cumulative in columns D and F). As an illustration, for year 3 the interest I3 and revenue plus interest R3 are

I_3 = (cumulative revenue through year 2) (rate of return)
= $416,000(0.08)
= $33,280
R_3 = revenue in year 3 + I_3
= $300,000 + 33,280
= $333,280

The detailed relations shown in Figure (1–10 *b)* calculate these values in cells C8 and E8.

Cell C8 relation for I_3 : = F7×B2

Cell E8 relation for CF_3 := B8 + C8

The equivalent amount after 4 years is $1,109,022, which is comprised of $1,000,000 in total revenue and $109,022 in interest compounded at 8% per year. The shaded cells in Figure (1–10 a) and b indicate that the sum of the annual values and the last entry in the cumulative columns must be equal.

2) To determine the effect of increasing estimated revenue for years 3 and 4 to $600,000, use the same spreadsheet and change the entries in cells B8 and B9 as shown in Figure (1–10 c). Total interest increases 22%, or $24,000, from $109,222 to $133,222.
3) Figure (1–10 d) shows the effect of changing the original i value from 8% to an inflation adjusted rate of 3.85% in cell B2 on the first spreadsheet. [Remember to return to the $300,000 revenue estimates for years 3 and 4 after working part (2).]

Inflation has now reduced total interest by 53% from $109,222 to $51,247, as shown in cell C10.

	A	B	C	D	E	F
1	Part (a) - Find totals in year 4					
2	i =	8.0%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	16,000	16,000	216,000	416,000
8	3	300,000	33,280	49,280	333,280	749,280
9	4	300,000	59,942	109,222	359,942	1,109,222
10			109,222		1,109,222	

Figure (1-10 a): Total interest and revenue for base case, year 4

	A	B	C	D	E	F
1	Part (a) - Find totals in year 4					
2	i =	0.08				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200000	0	=C6	=B6 + C6	=E6
7	2	200000	=F6*B1	=C7 + D6	=B7 + C7	=E7 + F6
8	3	300000	=F7*B1	=C8 + D7	=B8 + C8	=E8 + F7
9	4	300000	=F8*B1	=C9 + D8	=B9 + C9	=E9 + F8
10			=SUM(C6:C9)		=SUM(E6:E9)	

Figure (1-10 b): Spreadsheet relations for base case

	A	B	C	D	E	F
1	Part (b) - Find totals in year 4 with increased revenues					
2	i =	8.0%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	16,000	16,000	216,000	416,000
8	3	600,000	33,280	49,280	633,280	1,049,280
9	4	600,000	83,942	133,222	683,942	1,733,222
10			133,222		1,733,222	
11						
12			Revenue changed			
13						

Figure (1-10 c): Totals with increased revenue in years 3 and 4

	A	B	C	D	E	F
1	Part (c) - Find totals in year 4 considering 4% inflation					
2	i =	3.85%				
3–4	End of Year	Revenue at end of year, $	Interest earned during year, $	Cumulative interest, $	Revenue during year with interest, $	Cumulative revenue with interest, $
5	0					
6	1	200,000	0	0	200,000	200,000
7	2	200,000	7,700	7,700	207,700	407,700
8	3	300,000	15,696	23,396	315,696	723,396
9	4	300,000	27,851	51,247	327,851	1,051,247
10			51,247		1,051,247	

Rate of return changed

Figure (1-10 d): Totals with inflation of 4% per year considered

Problems

1- List the four essential elements involved in decision making in engineering economic analysis.
2- What is meant by (a) limited capital funds and (b) sensitivity analysis?
3- List three measures of worth that are used in engineering economic analysis.
4- Identify the following factors as either economic (tangible) or noneconomic (intangible): first cost, leadership, taxes, salvage value, morale, dependability, inflation, profit, acceptance, ethics, interest rate.
5- Emerson Processing borrowed $900,000 for installing energy-efficient lighting and safety equipment in its La Grange manufacturing facility. The terms of the loan were such that the company could pay interest only at the end of each year for up to 5 years, after which the company would have to pay the entire amount due. If the interest rate on the loan was 12% per year and the company paid only the interest for 4 years, determine the following:
   a) The amount of each of the four interest payments
   b) The amount of the final payment at the end of year 5
6- Which of the following 1-year investments has the highest rate of return?
   a) $12,500 that yields $1125 in interest,
   b) $56,000 that yields $6160 in interest, or
   c) $95,000 that yields $7600 in interest.
7- The symbol P represents an amount of money at a time designated as present. The following symbols also represent a present amount of money and require similar calculations. Explain what each symbol stands for: PW, PV, NPV, DCF, and CC
8- What is meant by end-of-period convention?
9- Construct a cash flow diagram to find the present worth in year 0 at an interest rate of 15% per year for the following situation.

Year	***Cash Flow, $***
0	-19,000
1–4	+8,100

10- Construct a cash flow diagram that represents the amount of money that will be accumulated in 15 years from an investment of $40,000 now at an interest rate of 8% per year.
11- At an interest rate of 15% per year, an investment of $100,000 one year ago is equivalent to how much now?
12- University tuition and fees can be paid by using one of two plans.
   Early-bird: Pay total amount due 1 year in advance and get a 10% discount.
   On-time: Pay total amount due when classes start.
   The cost of tuition and fees is $10,000 per year.
   a) How much is paid in the early-bird plan?
   b) What is the equivalent amount of the savings compared to the on-time payment at the time that the on-time payment is made?
13- If a company sets aside $1,000,000 now into a contingency fund, how much will the company have in 2 years, if it does not use any of the money and the account grows at a rate of 10% per year?
14- To finance a new product line, a company that makes high-temperature ball bearings borrowed $1.8 million at 10% per year interest. If the company repaid the loan in a lump sum amount after 2 years, what was:

a) The amount of the payment
b) The amount of interest?

15- If interest is compounded at 20% per year, how long will it take for \$50,000 to accumulate to \$86,400?

16- Give three other names for minimum attractive rate of return.

17- What is the weighted average cost of capital for a corporation that finances an expansion project using 30% retained earnings and 70% venture capital? Assume the interest rates are 8% for the equity financing and 13% for the debt financing.

18- State the purpose for each of the following built-in spreadsheet functions.
a) *PV (i% , n , A , F)*
b) *FV (i %, n , A , P)*
c) *RATE (n , A , P , F)*
d) *IRR (first_cell:last_cell)*
e) *PMT (i %, n , P , F)*
f) *NPER (i %, A , P , F)*

19- What are the values of the engineering economy symbols P , F , A , i , and n in the following functions? Use a question mark for the symbol that is to be determined.
a) *NPER (8%,-1500,8000,2000)*
b) *FV (7%,102000,-9000)*
c) *RATE (10,1000,-12000,2000)*
d) *PMT (11%,20,,14000)*
e) *PV (8%,15,-1000,800)*

Chapter 2

Factors: How Time and Interest Affect Money

Chapter 2
Factors: How Time and Interest Affect Money

General Objective:

Trainee will be able to understand the basic concepts and terminology necessary for engineering economy

Estimated time for this chapter: 5 Training hours

Detailed Objectives:

1. Single-Payment Factors (F/P and P/F).
2. Uniform Series Formulas (P/A, A/P, A/F, F/A).
3. Arithmetic Gradient Factors (P⁄G and A⁄G).
4. Geometric Gradient Series Factors.
5. Calculations for Cash Flows That Are Shifted.
6. Using Spreadsheets for Equivalency Computation.

Introduction

The cash flow is fundamental to every economic study. Cash flows occur in many configurations and amounts—isolated single values, series that are uniform, and series that increase or decrease by constant amounts or constant percentages. This chapter develops derivations for all the commonly used engineering economy factors that take the time value of money into account. The application of factors is illustrated using their mathematical forms and a standard notation format. Spreadsheet functions are used in order to rapidly work with cash flow series and to perform sensitivity analysis.

2.1. Single-Amount Factors (F / P and P / F)

The ***most fundamental factor in engineering economy*** is the one that determines the amount of money *F* accumulated after *n* years (or periods) from a single present worth *P*, with interest compounded one time per year (or period). Recall that compound interest refers to interest paid on top of interest. Therefore, if an amount P is invested at time $t = 0$, the amount F_1 accumulated 1 year hence at an interest rate of *i* percent per year will be

$$F_1 = P + Pi = P(1+i)$$

where the interest rate is expressed in decimal form. At the end of the second year, the amount accumulated F_2 is the amount after year 1 plus the interest from the end of year 1 to the end of year 2 on the entire F_1.

$$F_2 = F_1 + F_1 i = P(1+i) + P(1+i)i \qquad (2-1)$$

The amount F_2 can be expressed as

$$F_2 = P(1+i+i+i^2) = P(1+2i+i^2) = P(1+i)^2$$

Similarly, the amount of money accumulated at the end of year 3, using Equation (2-1), will be

$$F_3 = F_2 + F_2 i$$

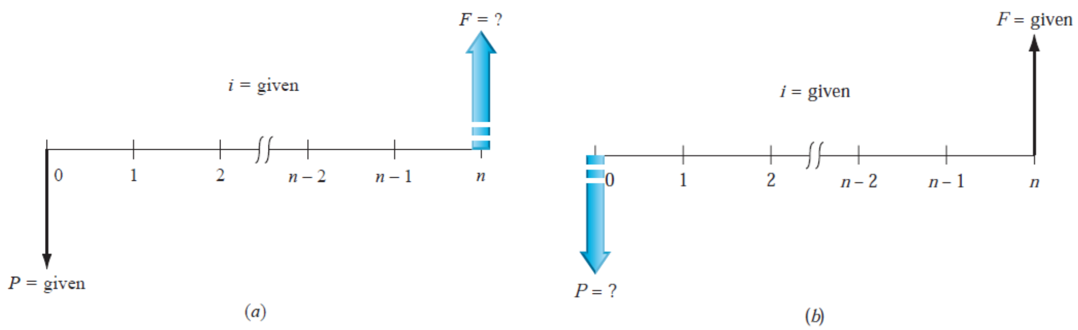


Figure (2-1): Cash flow diagrams for single-payment factors: (a) find F, given P, and (b) find P, given F.

From the preceding values, it is evident by mathematical induction that the formula can be generalized for *n* years. To find *F*, given *P*,

$$F = P(1+i)^n \qquad (2-2)$$

The factor $(\mathbf{1}+\boldsymbol{i})^{\boldsymbol{n}}$ is called the ***single-payment compound amount factor (SPCAF)***, but it is usually referred to as the ***F/P factor***. This is the conversion factor that, when multiplied by *P,* yields the future amount *F* of an initial amount *P* after *n* years at interest rate *i.* The cash flow diagram is seen in Figure (2–1 a).

Reverse the situation to ***determine the P value for a stated amount F*** that occurs *n* periods in the future. Simply solve Equation (2-2) for *P*.

$$P = F\left[\frac{1}{(1+i)^n}\right] = F(1+i)^{-n} \qquad (2-3)$$

The expression $(\mathbf{1}+\boldsymbol{i})^{-\boldsymbol{n}}$ is known as the ***single-payment present worth factor (SPPWF)***, or the ***P/F factor***. This expression determines the present worth *P* of a given future amount *F* after *n* years at interest rate *i*. The cash flow diagram is shown in Figure (2–1 b).

Note that the two factors derived here are for *single payments;* that is, they are used to find the present or future amount when only one payment or receipt is involved.

A standard notation has been adopted for all factors. The notation includes two cash flow symbols, the interest rate, and the number of periods. It is always in the general form (X/Y, i, n). The letter X represents what is sought, while the letter Y represents what is given. For example, F/P means find F when given P. The i is the interest rate in percent, and n represents the number of periods involved.

Using this notation, (F/*P* ,6%,20) represents the factor that is used to calculate the future amount *F* accumulated in 20 periods if the interest rate is 6% per period. The *P* is given. The standard notation, simpler to use than formulas and factor names, will be used hereafter.

Table (2–1) summarizes the standard notation and equations for the *F*/*P* and *P*/*F* factors.

Table (2–1): F/P and P/F Factors: Notation and Equations

Factor Notation	Factor Name	Find/Given	Standard Notation Equation	Equation with Factor Formula	Excel Function
$(F/P, i, n)$	Single-payment compound amount	F/P	$F = P(F/P, i, n)$	$F = P(1+i)^n$	$= FV(i\%,n,,P)$
$(P/F, i, n)$	Single-payment present worth	P/F	$P = F(P/F, i, n)$	$P = F(1+i)^{-n}$	$= PV(i\%,n,,F)$

For **spreadsheets**, a future value *F* is calculated by the FV function using the format

$$= FV(i\%, n,, P) \qquad (2-4)$$

A present amount *P* is determined using the PV function with the format

$$= PV(i\%, n,, F) \qquad (2-5)$$

Example (2-1)

Sandy, a manufacturing engineer, just received a year-end bonus of $10,000 that will be invested immediately. With the expectation of earning at the rate of 8% per year, Sandy hopes to take the entire amount out in exactly 20 years to pay for a family vacation when the oldest daughter is due to graduate from college. Find the amount of funds that will be available in 20 years by using:

1) hand solution by applying the factor formula and tabulated value
2) a spreadsheet functions.

Solution

The cash flow diagram is the same as Figure (2–1*a*). The symbols and values are

$P = \$10{,}000 \quad F = ? \quad i = 8\%$ per year $\quad n = 20$ years

1) Factor formula: Apply Equation (2-2) to find the future value *F*. Rounding to four decimals, we have

$$F = P(1+i)^n = 10{,}000(1.08)^{20} = 10{,}000(4.6610) = \$46{,}610$$

Standard notation and tabulated value: Notation for the *F*/*P* factor is (*F*/*P*,*i*%,*n*).

$$F = P(F/P, 8\%, 20) = 10{,}000(4.6610) = \$46{,}610$$

2) Spreadsheet: Use the FV function to find the amount 20 years in the future. The format is that shown in Equation (2-4); the numerical entry is = *FV(8%,20,,10000).*

2.2. Uniform Series Formulas (P/A, A/P, A/F, F/A)

The equivalent present worth *P* of a uniform series *A* of end-of-period cash flows (investments) is shown in Figure (2–2 a). An expression for the present worth can be determined by considering each *A* value as a future worth *F,* calculating its present worth with the *P*/*F* factor, Equation (2-3), and summing the results.

$$P = A\left[\frac{1}{(1+i)^1}\right] + A\left[\frac{1}{(1+i)^2}\right] + \cdots + A\left[\frac{1}{(1+i)^{n-1}}\right] + A\left[\frac{1}{(1+i)^n}\right] \quad (2-6)$$

The terms in brackets are the *P*/*F* factors for years 1 through *n,* respectively. Factor out *A*.

To simplify Equation (2-6) and obtain the *P*/*A* factor, multiply the *n* -term geometric progression in brackets by the *(P/F, i %,1)* factor, which is *1/(1 +i)*. Simplify to obtain the expression for P when i ≠ 0 (Equation (2-7)).

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] ; i \neq 0 \qquad (2-7)$$

The term in brackets in Equation (2-7) is the conversion factor referred to as the *uniform series present worth factor* (USPWF). It is the ***P*/*A* factor** used to calculate the ***equivalent P value in year 0*** for a uniform end-of-period series of *A* values beginning at the end of period 1 and extending for *n* periods. The cash flow diagram is Figure (2–2 a).

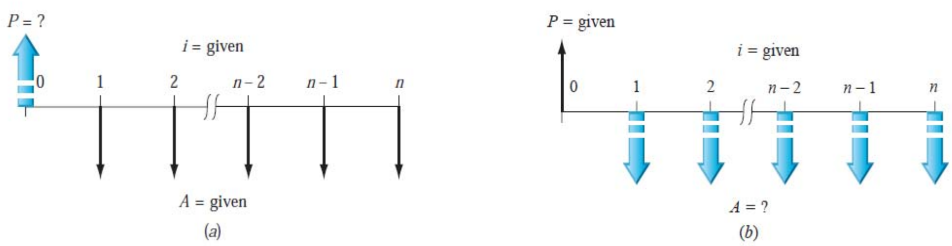


Figure (2-2): Cash flow diagrams used to determine: (a) P, given a uniform series A, and (b) A, given a present worth P**.**

To reverse the situation, the present worth *P* is known and the equivalent uniform series amount *A* is sought (Figure (2–4 b)). The first *A* value occurs at the end of period 1, that is, one period after *P* occurs. Solve Equation (2-7) for *A* to obtain

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad (2-8)$$

The term in brackets is called the *capital recovery factor* (CRF), or ***A*/*P* factor**. It calculates the ***equivalent uniform annual worth A*** over *n* years for a given *P* in year 0, when the interest rate is *i*.

The P/A and A/P factors are derived with the present worth P and the first uniform annual amount A one year (period) apart. That is, the present worth P must always be located one period prior to the first A

Spreadsheet functions can determine both P and A values in lieu of applying the P/A and A/P factors. The PV function calculates the P value for a given A over n years and a separate F value in year n, if it is given. The format, is

$$= PV(\, i\,\%, n\,, A\,, F\,) \qquad (2-9)$$

Similarly, the A value is determined by using the PMT function for a given P value in year 0 and a separate F, if given. The format is

$$= \; PMT(\, i\,\%, n\,, P\,, F\,) \qquad (2-10)$$

The simplest way to derive the A/F factor is to substitute into factors already developed. If P from Equation (2-3) is substituted into Equation (2-8), the following formula results.

$$A = \; F\left[\frac{1}{(1+i)^n}\right]\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] \; ; \;\; A = \; F\left[\frac{i}{(1+i)^n-1}\right] \qquad (2-11)$$

The expression in brackets in Equation (2-11) is the ***A/F*** **or sinking fund factor**. It determines the **uniform annual series** ***A*** that is equivalent to a given future amount F. This is shown graphically in Figure (2–3 a), where A is a uniform annual investment.

The uniform series A begins at the end of year (period) 1 and continues through the year of the given F. The last A value and F occur at the same time.

Equation (2-11) can be rearranged to find F for a stated A series in periods 1 through n (Figure 2–3 b).

$$F = \; A\left[\frac{(1+i)^n-1}{i}\right] \qquad (2-12)$$

For solution by **spreadsheet**, the FV function calculates F for a stated A series over n years. The format is

$$= FV(\, i\,\%, n\,, A\,, P\,) \qquad (2-13)$$

The P may be omitted when no separate present worth value is given. The PMT function determines the A value for n years, given F in year n and possibly a separate P value in year 0. The format is

$$= \; PMT(\, i\,\%, n\,, P\,, F\,) \qquad (2-14)$$

If P is omitted, the comma must be entered so the function knows the last entry is an F value.

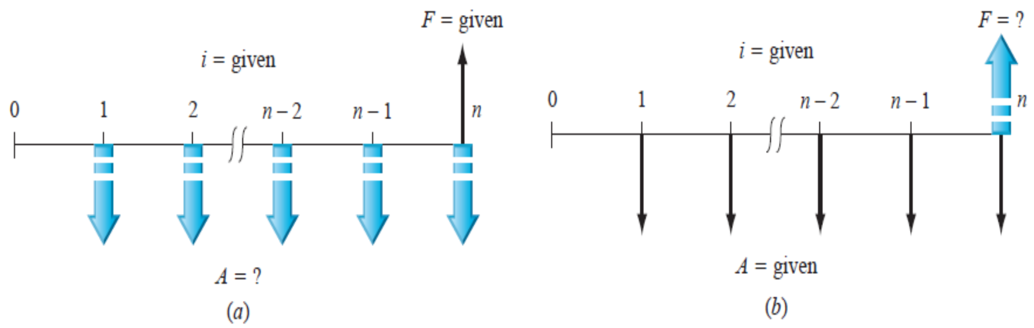


Figure (2-3): Cash flow diagrams to (a) find A, given F, and (b) find F, given A.

Table (2–2) summarizes the standard notation and equations for the *A*/*P*, *P*/*A*, *F*/*A* and *A*/*F* factors.

Table (2-2): P/A, A/P, A/F and F/A Factors: Notation and Equations

Notation	Factor Name	Find/Given	Factor Formula	Standard Notation Equation	Excel Function
$(P/A, i, n)$	Uniform series present worth	P/A	$\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$	$P = A(P/A, i, n)$	$= PV(i\%,n,A)$
$(A/P, i, n)$	Capital recovery	A/P	$\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$	$A = P(A/P, i, n)$	$= PMT(i\%,n,P)$
$(F/A, i, n)$	Uniform series compound amount	F/A	$\left[\frac{(1+i)^n-1}{i}\right]$	$F = A(F/A, i, n)$	$= FV(i\%,n,A)$
$(A/F, i, n)$	Sinking fund	A/F	$\left[\frac{i}{(1+i)^n-1}\right]$	$A = F(A/F, i, n)$	$= PMT(i\%,n,F)$

Example (2-2)

How much money should you be willing to pay now for a guaranteed \$600 per year for 9 years starting next year, at a rate of return of 16% per year?

Solution

The cash flows follow the pattern of Figure 2–2 *a* , with A = \$600, i = 16%, and n = 9. The present worth is

$$P = 600(P/A, 16\%, 9) = 600(4.6065) = \$2763.90$$

The PV function = *PV(16%,9,600)* entered into a single spreadsheet cell will display the answer P = (\$2763.93).

Example (2-3)

The president of Ford Motor Company wants to know the equivalent future worth of a \$1 million capital investment each year for 8 years, starting 1 year from now. Ford capital earns at a rate of 14% per year.

Solution

The cash flow diagram (Figure (2–4)) shows the annual investments starting at the end of year 1 and ending in the year the future worth is desired. In \$1000 units, the F value in year 8 is found by using the F/A factor.

$$F = 1000(F/A, 14\%, 8) = 1000(13.2328) = \$13,232.80$$

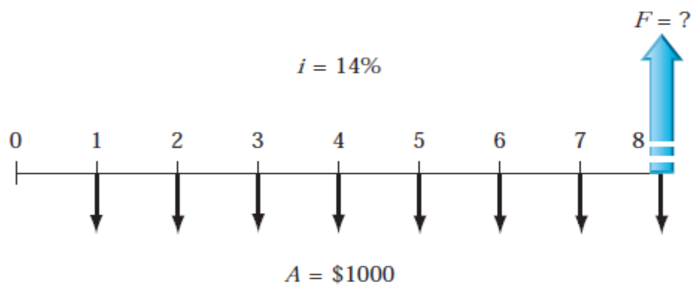


Figure (2–4): Diagram to find F for a uniform series, Example (2-3).

2.3. Arithmetic Gradient Factors (P⁄G and A⁄G)

Assume a manufacturing engineer predicts that the cost of maintaining a robot will increase by $5000 per year until the machine is retired. The cash flow series of maintenance costs involves a constant gradient, which is $5000 per year.

An arithmetic gradient series is a cash flow series that either increases or decreases by a constant amount each period. The amount of change is called the gradient.

Formulas previously developed for an *A* series have year-end amounts of equal value. In the case of a gradient, each year-end cash flow is different, so new formulas must be derived. First, assume that the cash flow at the end of year 1 is the **base amount** of the cash flow series and, therefore, not part of the gradient series. This is convenient because in actual applications, the base amount is usually significantly different in size compared to the gradient. For example, if you purchase a used car with a 1-year warranty, you might expect to pay the gasoline and insurance costs during the first year of operation. Assume these costs $2500; that is, $2500 is the base amount. After the first year, you absorb the cost of repairs, which can be expected to increase each year. If you estimate that total costs will increase by $200 each year, the amount the second year is $2700, the third $2900, and so on to year *n*, when the total cost is 2500 + (*n* - 1)200. The cash flow diagram is shown in Figure (2–5). Note that the gradient ($200) is first observed between year 1 and year 2, and the base amount ($2500 in year 1) is not equal to the gradient.

Define the symbols *G* for gradient and CF_n for cash flow in year n as follows.

G = constant arithmetic change in cash flows from one time period to the next; G may be positive or negative.

$$CF_n = base\ amount + (n-1)\,G \qquad (2-15)$$

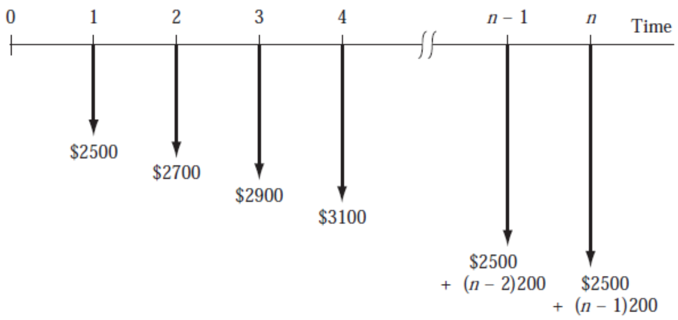


Figure (2–5): Cash flow diagram of an arithmetic gradient series.

It is important to realize that the base amount defines a uniform cash flow series of the size *A* that occurs each time period. We will use this fact when calculating equivalent amounts that involve arithmetic gradients. If the base amount is ignored, a generalized arithmetic (increasing) gradient cash flow diagram is as shown in Figure (2–6). Note that the gradient begins between years 1 and 2. This is called a **conventional gradient.**

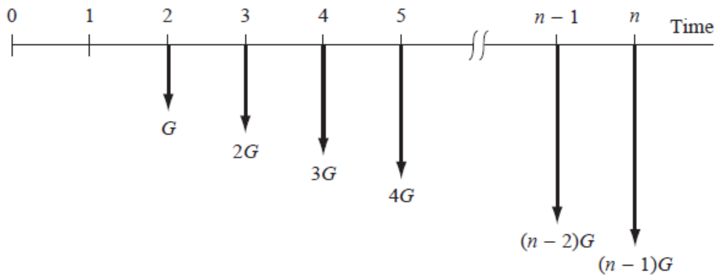


Figure (2–6): Conventional arithmetic gradient series without the base amount.

Example (2-4)

A local university has initiated a logo-licensing program with the clothier Holister, Inc. Estimated fees (revenues) are $80,000 for the first year with uniform increases to a total of $200,000 by the end of year 9. Determine the gradient and construct a cash flow diagram that identifies the base amount and the gradient series.

Solution

The year 1 base amount is CF_1 = $80,000, and the total increase over 9 years is

$$CF_9 - CF_1 = 200{,}000 - 80{,}000 = \$120{,}000$$

Equation (2-15), solved for *G,* determines the arithmetic gradient.

$$G = \left[\frac{CF_9 - CF_1}{n-1}\right] = \left[\frac{\$120{,}000}{9-1}\right] = \$15{,}000 \text{ per year}$$

The cash flow diagram Figure (2–7) shows the base amount of $80,000 in years 1 through 9 and the $15,000 gradient starting in year 2 and continuing through year 9.

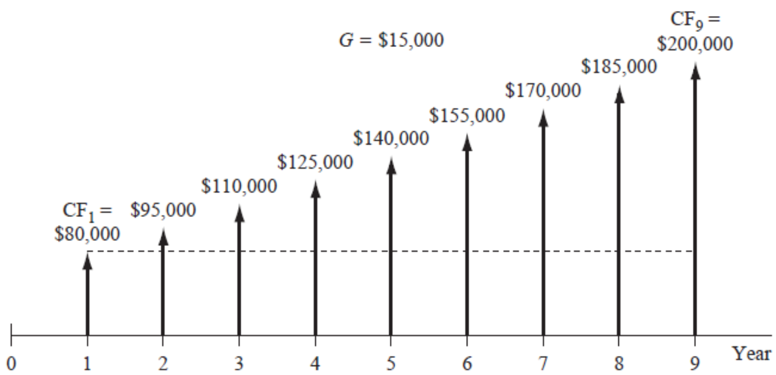


Figure (2–7): Diagram for gradient series, Example (2-4).

The **total present worth** $\boldsymbol{P_T}$ for a series that includes a base amount *A* and conventional arithmetic gradient must consider the present worth of both the uniform series defined by *A* and the arithmetic gradient series. The addition of the two results in P_T.

$$P_T = P_A \pm P_G \qquad (2-16)$$

where P_A is the present worth of the uniform series only, P_G is the present worth of the gradient series only, and the – or + sign is used for an increasing (*–G)* or decreasing (+*G)* gradient, respectively.

The corresponding equivalent annual worth A_T is the sum of the base amount series annual worth A_A and gradient series annual worth A_G, that is,

$$A_T = A_A \pm A_G \qquad (2-17)$$

Three factors are derived for arithmetic gradients: The *P*/*G* factor for present worth, the *A*/*G* factor for annual series, and the *F*/*G* factor for future worth. There are several ways to derive them. We use the single-payment present worth factor (P/*F*, *i*, *n*), but the same result can be obtained by using the *F*/*P* , *F*/*A* , or *P*/*A* factor.

In Figure (2–6), the present worth at year 0 of only the gradient is equal to the sum of the present worth's of the individual cash flows, where each value is considered a future amount.

$$P = G(P/F,i,2) + 2G(P/F,i,3) + 3G(P/F,i,4) + \cdots + [(n-2)G]\big(P/F,i,(n-1)\big) + [(n-1)G](P/F,i,n)$$

Factor out *G* and use the *P*/*F* formula.

$$P = G\left[\frac{1}{(1+i)^2} + \frac{2}{(1+i)^3} + \frac{3}{(1+i)^4} + \cdots + \frac{n-2}{(1+i)^{n-1}} + \frac{n-1}{(1+i)^n}\right] \qquad (2-18)$$

Multiplying both sides of Equation (2-18) by $(1+i)^1$ yields

$$P(1+i)^1 = G\left[\frac{1}{(1+i)^1} + \frac{2}{(1+i)^2} + \frac{3}{(1+i)^3} + \cdots + \frac{n-2}{(1+i)^{n-2}} + \frac{n-1}{(1+i)^{n-1}}\right] \quad (2-19)$$

Subtract Equation (2-18) from Equation (2-19) and simplify.

$$iP = G\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] - G\left[\frac{n}{(1+i)^n}\right] (2-20)$$

The left bracketed expression is the same as that contained in Equation (2-6), where the *P*/*A* factor was derived. Substitute the closed-end form of the *P*/*A* factor from Equation (2-7).

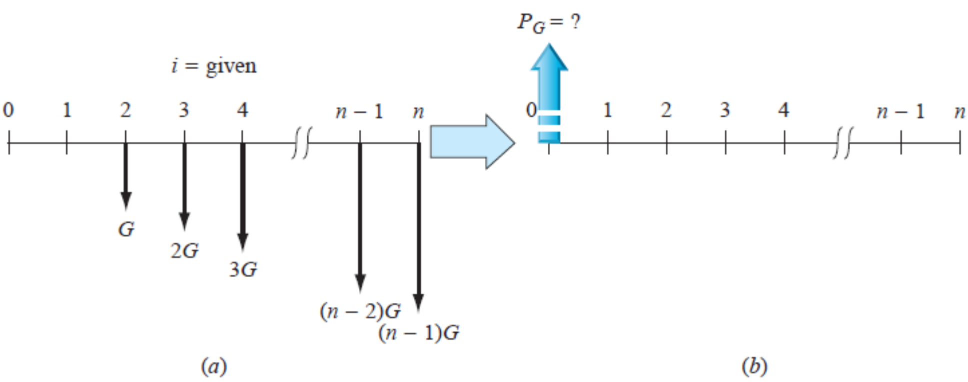


Figure (2–8): Conversion diagram from an arithmetic gradient to a present worth.

into Equation (2-20) and simplify to solve for P_G, the present worth of the gradient series only.

$$P_G = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right] \qquad (2-21)$$

Equation (2-21) is the general relation to **convert an arithmetic gradient *G* (not including the base amount) for *n* years into a present worth at year 0.** Figure (2–8 *a*) is converted into the equivalent cash flow in Figure (2–8 *b*). The *arithmetic gradient present worth factor,* or **P/G factor,** may be expressed in two forms:

$$(\mathrm{P/G}, i, n) = \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right] \; or \; (\mathrm{P/G}, i, n) = \frac{(1+i)^n - in - 1}{i^2(1+i)^n} \quad (2-22)$$

Remember: The conventional arithmetic gradient starts in year 2, and P is located in year 0.

Equation (2-21) expressed as an engineering economy relation is

$$P_G = G(\mathrm{P/G}, i, n) \qquad (2-23)$$

which is the rightmost term in Equation (2-16) to calculate total present worth. The *G* carries a minus sign for decreasing gradients.

The equivalent uniform annual series A_G for an arithmetic gradient *G* is found by multiplying the present worth in Equation (2-23) by the (A/*P, i, n)* formula. In standard notation form, the equivalent of algebraic cancellation of *P* can be used.

$$A_G = G(P/G, i, n)(A/P, i, n) = G(A/G, i, n)$$

In equation form,

$$A_G = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$A_G = G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] \qquad (2-24)$$

which is the rightmost term in Equation (2-17). The expression in brackets in Equation (2-24) is called the *arithmetic gradient uniform series factor* and is identified by ***(A/ G, i, n)***. This factor converts Figure (2–9 a) into Figure (2–9 b).

The *P*/*G* and *A*/*G* factors and relations are summarized inside the front cover. Factor values are tabulated in the two rightmost columns of factor values at the rear of this text.

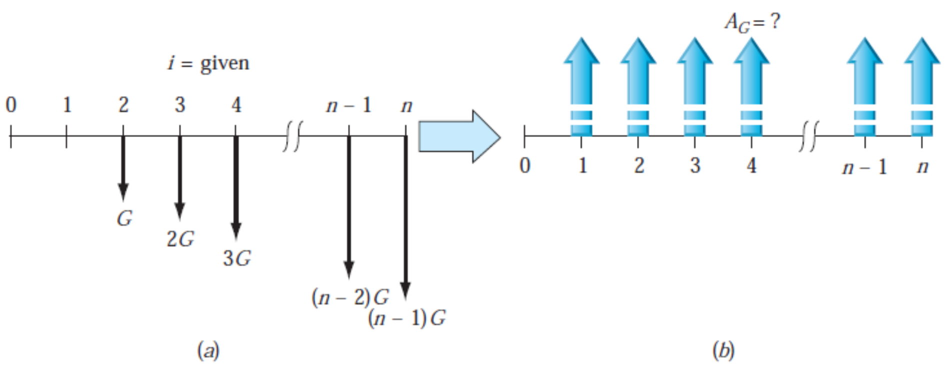


Figure (2–9): Conversion diagram of an arithmetic gradient series to an equivalent uniform annual series.

There is no direct, single-cell spreadsheet function to calculate P_G or A_G for an arithmetic gradient. Use the NPV function to display P_G and the PMT function to display A_G after entering all cash flows (base and gradient amounts) into contiguous cells. General formats for these functions are:

$$= NPV(i\%, second_{cell}: last_{cell}) + first_{cell} \qquad (2-25)$$

$$= PMT(i\%, n, cell\_with\_P_G) \qquad (2-26)$$

An ***F/G* factor** (arithmetic *gradient future worth factor)* to calculate the future worth *F G* of a gradient series can be derived by multiplying the *P/G* and *F/P* factors. The resulting factor, (F/*G, i, n)*, in brackets, and engineering economy relation is

$$F_G = G\left[\left(\frac{1}{i}\right)\left(\frac{(1+i)^n - 1}{i}\right) - n\right]$$

Example (2-5)

Neighboring parishes in Louisiana have agreed to pool road tax resources already designated for bridge refurbishment. At a recent meeting, the engineers estimated that a total of $500,000 will be deposited at the end of next year into an account for the repair of old and safety-questionable bridges throughout the area. Further, they estimate that the deposits will increase by $100,000 per year for only 9 years thereafter, then cease.

1) Determine the equivalent present worth
2) Determine the equivalent annual series amounts, if public funds earn at a rate of 5% per year.

Solution

1) The cash flow diagram of this conventional arithmetic gradient series from the perspective of the parishes is shown in Figure (2–10). According to Equation (2-16), two computations must be made and added: the first for the present worth of the base amount P_A and the second for the present worth of the gradient P_G. The total present worth P_T occurs in year 0. This is illustrated by the partitioned cash flow diagram in Figure (2–11). In $1000 units, the total present worth is

$$\begin{aligned} P_T &= 500(P/A, 5\%, 10) + 100(P/G, 5\%, 10) \\ &= 500(7.7217) + 100(31.6520) \\ &= \$7026.05\ (\$7{,}026{,}050) \end{aligned}$$

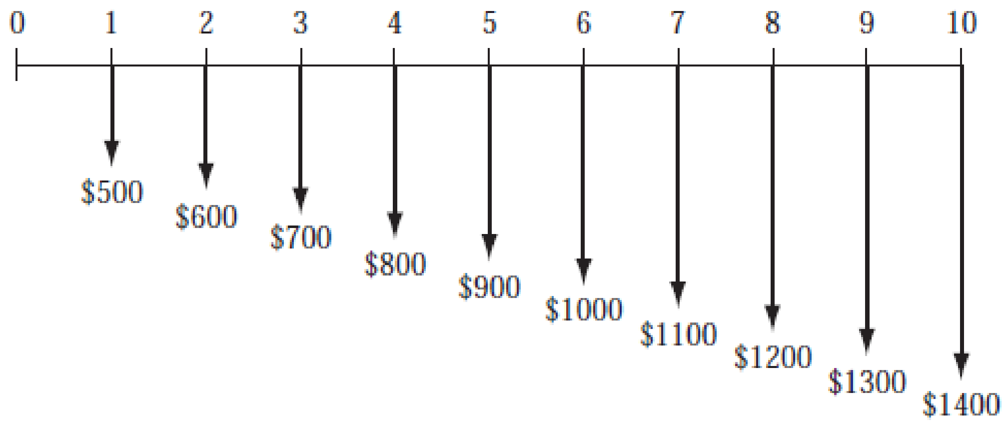


Figure (2–10): Cash flow series with a conventional arithmetic gradient (in $1000 units), Example (2-5).

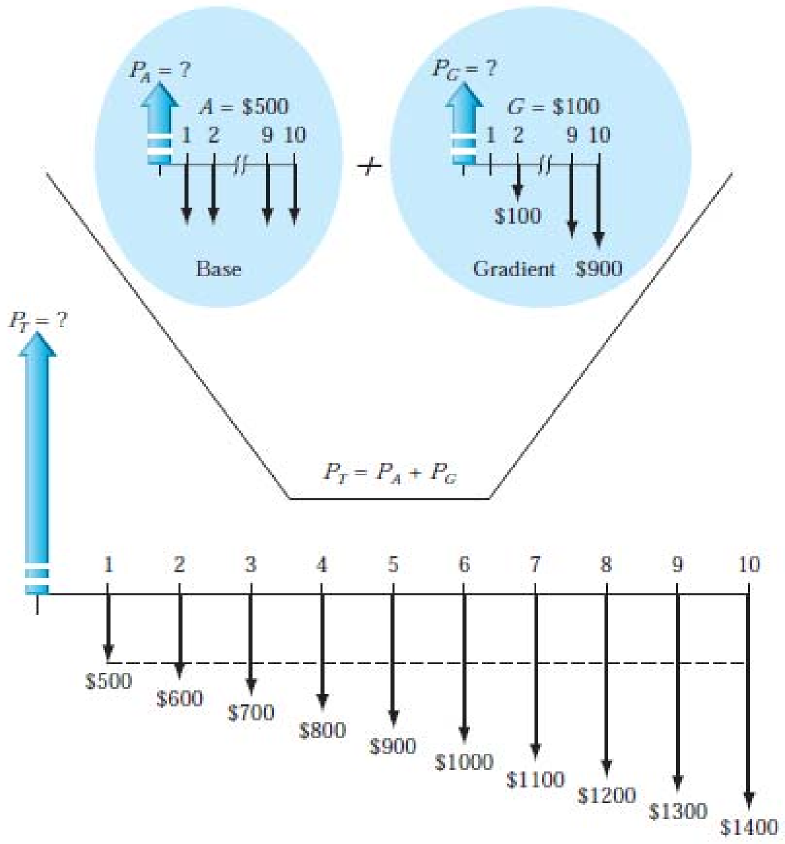


Figure (2–11): Partitioned cash flow diagram (in $1000 units), Example (2-5).

2) Here, too, it is necessary to consider the gradient and the base amount separately. The total annual series A_T is found by Equation (2-17) and occurs in years 1 through 10.

$$A_T = 500 + 100(A/G, 5\%, 10) = 500 + 100(4.0991)$$
$$= \$909.91 \text{ per year} \quad (\$909.910)$$

2.4. Geometric Gradient Series Factors

It is common for annual revenues and annual costs such as maintenance, operations, and labor to go up or down by a constant percentage, for example, +5% or -3% per year. This change occurs every year on top of a starting amount in the first year of the project. A definition and description of new terms follow.

A geometric gradient series is a cash flow series that either increases or decreases by a constant percentage each period. The uniform change is called the rate of change.

- ***g*** = constant rate of change, in decimal form, by which cash flow values increase or decrease from one period to the next. The gradient g can be + or -.
- $\boldsymbol{A_1}$= initial cash flow in year 1 of the geometric series
- $\boldsymbol{P_g}$ = present worth of the entire geometric gradient series, including the initial amount A_1

Note that the initial cash flow A_1 is not considered separately when working with geometric gradients. Figure (2–12) shows increasing and decreasing geometric gradients starting at an amount A_1 in time period 1 with present worth P_g located at time 0. The relation to determine the total present worth P_g for the entire cash flow series may be derived by multiplying each cash flow in Figure (2–12 a) by the *P/F* factor *1/ (1 + i)ⁿ*.

$$P_g = \frac{A_1}{(1+i)^1} + \frac{A_1(1+g)}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \cdots + \frac{A_1(1+g)^{n-1}}{(1+i)^n}$$

$$= A_1\left[\frac{1}{(1+i)^1} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \cdots + \frac{(1+g)^{n-1}}{(1+i)^n}\right] \quad (2-27)$$

Multiply both sides by (1 + *g*)/ (1 + *i*), subtract Equation (2-27) from the result, factor out P_g, and obtain

$$P_g\left(\frac{(1+g)}{(1+i)} - 1\right) = A_1\left[\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{1+i}\right]$$

Solve for P_g and simplify.

$$P_g = A_1\left[\frac{1-(\frac{1+g}{1+i})^n}{i-g}\right] \;;\; g \neq i \qquad (2-28)$$

The term in brackets in Equation (2-27) is the ***(P/A, g, i, n)*** or *geometric gradient series present worth factor* for values of *g* not equal to the interest rate *i*. When *g = i,* substitute *i* for *g* in Equation (2-28) and observe that the term 1/ (1 + *i*) appears *n* times.

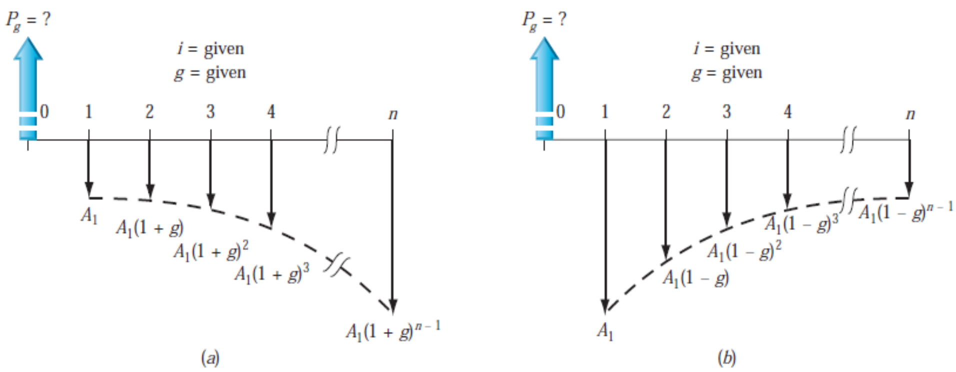


Figure (2–12): Cash flow diagram of (a) increasing and (b) decreasing geometric gradient series and present worth P_g.

$$P_g = A_1\left[\frac{1}{(1+i)} + \frac{1}{(1+i)} + \frac{1}{(1+i)} + \cdots + \frac{1}{(1+i)}\right]$$

$$P_g = \frac{nA_1}{(1+i)} \qquad (2-29)$$

The (P/A, g, i, n) factor calculates P_g in period t = 0 for a geometric gradient series starting in period 1 in the amount A_1 and increasing by a constant rate of g each period.

The equation for P_g and the $(P/A, g, i, n)$ factor formula are

$$P_g = A_1(P/A, g, i, n) \qquad (2-30)$$

$$(P/A, g, i, n) = \begin{cases} \dfrac{1-(\frac{1+g}{1+i})^n}{i-g} & ; \ g \neq i \\ \dfrac{n}{(1+i)} & ; \ g = i \end{cases} \qquad (2-31)$$

It is possible to derive factors for the equivalent A and F values; however, it is easier to determine the P_g amount and then multiply by the A/P or F/P factor. As with the arithmetic gradient series, there are no direct spreadsheet functions for geometric gradient series. Once the cash flows are entered, P and A are determined using the NPV and PMT functions, respectively.

Example (2-6)

A coal-fi red power plant has upgraded an emission control valve. The modification costs only \$8000 and is expected to last 6 years with a \$200 salvage value. The maintenance cost is expected to be high at \$1700 the first year, increasing by 11% per year thereafter. Determine the equivalent present worth of the modification and maintenance cost by hand and by spreadsheet at 8% per year.

Solution

1) Solution by Hand

The cash flow diagram Figure (2–13) shows the salvage value as a positive cash flow and all costs as negative. Use Equation (2-35) for $g \neq i$ to calculate P_g. Total P_T is the sum of three present worth components.

$$P_T = -8000 - P_g + 200(P/F, 8\%, 6)$$

$$= -8000 - \left[\frac{1-\left(\frac{1.11}{1.08}\right)^6}{0.08-0.11}\right] + 200(P/F, 8\%, 6)$$

$$= -8000 - 1700(5.9559) + 120 = \$-17{,}999$$

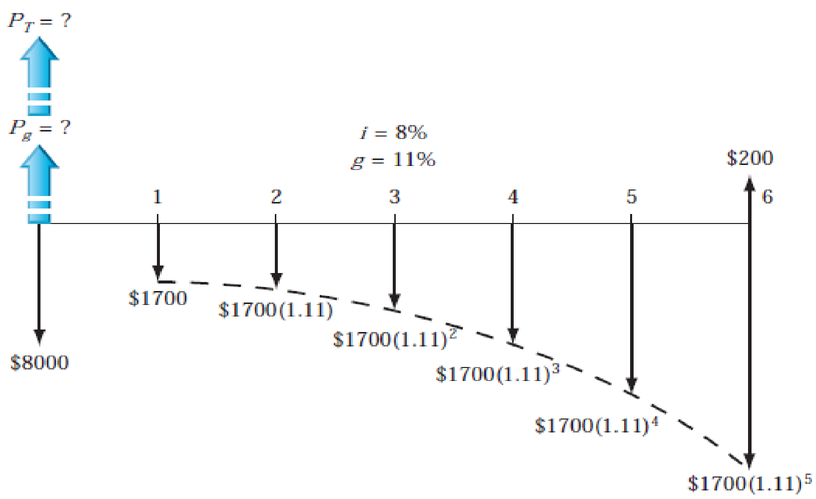


Figure (2–13): Cash flow diagram of a geometric gradient, Example (2-6).

2) Solution by Spreadsheet

Figure (2–14) details the spreadsheet operations to find the geometric gradient present worth P_g and total present worth P_T. To obtain P_T = \$-17,999, three components are summed—first cost, present worth of estimated salvage in year 6, and P_g. Cell tags detail the relations for the second and third components; the first cost occurs at time 0.

The relation that calculates the (*P*/*A*, *g*, *i*%, *n*) factor is rather complex, as shown in the cell tag and formula bar for C9. If this factor is used repeatedly, it is worthwhile using cell reference formatting so that A_1, *i*, *g*, and *n* values can be changed and the correct value is always obtained. Try to write the relation for cell C9 in this format.

C9 = -1700 * ((1- ((1.11)/(1.08))^6)/(0.08-0.11))

	A	B	C
1			
2	**Information provided**	**Estimates**	**P value, $**
3	Interest rate, i%	8%	
4	First cost, $	-8000	-8000
5			
6	Life, n, years	6	
7	Salvage, $	200	126
8			
9	Maintenance cost, year 1, $	-1,700	-10,125
10	Cost gradient, g%	11%	
11	**Total, $**		**-17,999**
12			

Present worth of salvage = −PV(8%,6,,200)

Present worth of maintenance costs, Eq. [2.35] = −1700* ((1-((1.11)/(1.08))^6)/(0.08-0.11))

Figure (2–14): Geometric gradient and total present worth calculated via spreadsheet, Example (2-6).

2.5. Calculations for Cash Flows That Are Shifted

When a uniform series begins at a time other than at the end of period 1, it is called a shifted series. In this case several methods based on factor equations or tabulated values can be used to find the equivalent present worth *P*.

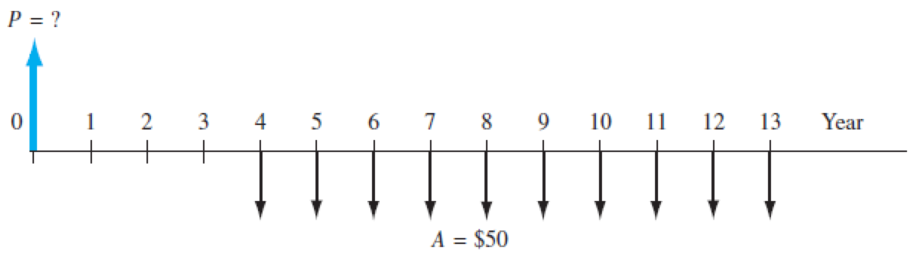


Figure (2–15): A uniform series that is shifted.

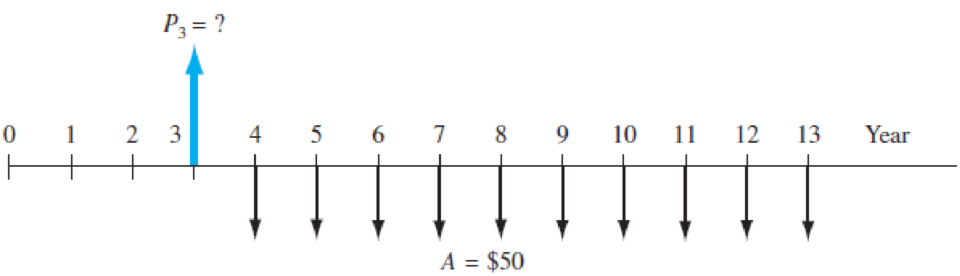


Figure (2–16): Location of present worth for the shifted uniform series in Figure (2–15)

For example, P of the uniform series shown in Figure (2-15) could be determined by any of the following methods:

- Use the *P/F* factor to find the present worth of each disbursement at year 0 and add them.
- Use the *F/P* factor to find the future worth of each disbursement in year 13, add them, and then find the present worth of the total using *P=F (P/F, i, 13).*
- Use the *F/A* factor to find the future amount *F=A (F/A, i, 10)*, and then compute the present worth using *P = F (P/F, i, 3).*
- Use the *P/A* factor to compute the "present worth" (which will be located in year 3 not year 0), and then find the present worth in year 0 by using the *(P/F, i, 3)* factor. (Present worth is enclosed in quotation marks here only to represent the present worth as determined by the *P/A* factor in year 3, and to differentiate it from the present worth in year 0.)

Typically, the last method is used. For Figure (2-15), the "present worth" obtained using the *P/A* factor is located in year 3. This is shown as P_3 in Figure (2-16).

Remember, the present worth is always located one period prior to the first uniform-series amount when using the P/A factor.

To determine a future worth or *F* value, recall that the *F/A* factor has the *F* located in the *same* period as the last uniform-series amount. Figure (2-17) shows the location of the future worth when *F/A* is used for Figure (2-15) cash flows.

Remember, the future worth is always located in the same period as the last uniform-series amount when using the F/A factor.

It is also important to remember that the number of periods *n* in the *P/A* or *F/A* factor is equal to the number of uniform-series values. It may be helpful to *renumber* the cash flow diagram to avoid errors in counting. Figure (2-17) shows Figure (2-15) renumbered to determine *n* = 10.

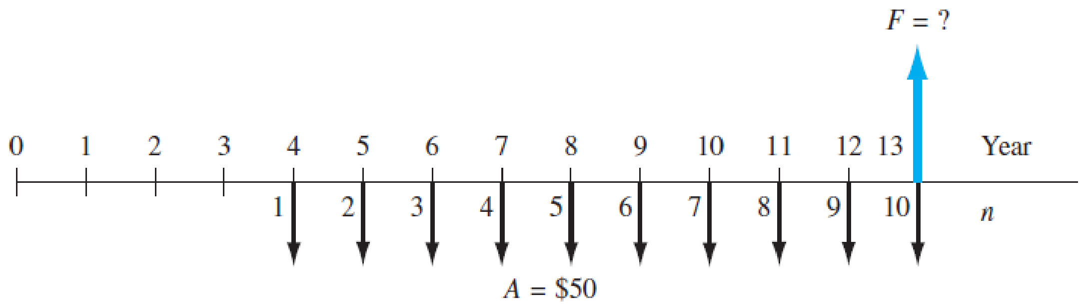


Figure (2–17): Placement of F and renumbering for n for the shifted uniform series of Figure (2-15)

As stated above, there are several methods that can be used to solve problems containing a uniform series that is shifted. However, it is generally more convenient to use the uniform-series factors than the single-amount factors. There are specific steps that should be followed in order to avoid errors:

1. Draw a diagram of the positive and negative cash flows.
2. Locate the present worth or future worth of each series on the cash flow diagram.
3. Determine *n* for each series by renumbering the cash flow diagram.

4. Set up and solve the equations.

Example (2-7)

An engineering technology group just purchased new CAD software for $5000 now and annual payments of $500 per year for 6 years starting 3 years from now for annual upgrades. What is the present worth of the payments if the interest rate is 8% per year?

Solution

The cash flow diagram is shown in Figure (2-18). The symbol P_A is used to represent the present worth of a uniform annual series *A*, and P_A' represents the present worth at a time other than period 0. Similarly, P_T represents the total present worth at time 0. The correct placement of P_A' and the diagram renumbering to obtain *n* are also indicated.

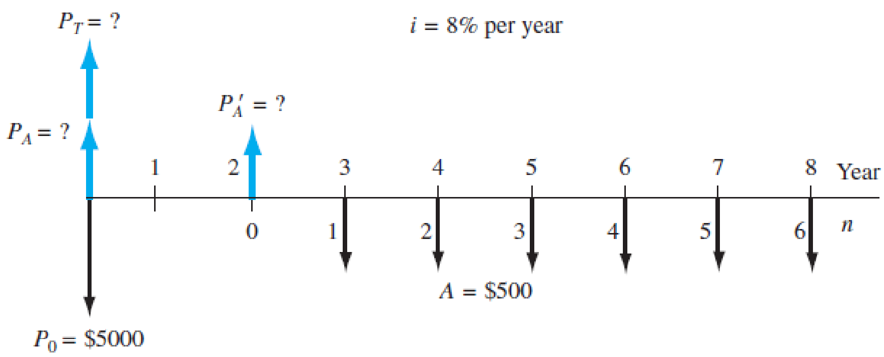


Figure (2–18): Cash flow diagram with placement of *P* values, Example (2-6)

Note that P_A' is located in actual year 2, not year 3. Also, $n = 6$ not 8, for the *P/A* factor. First find the value of P_A' the shifted series.

$$P_A' = \$500(P/A, 8\%, 6)$$

Since P_A' is located in year 2, now find P_A in year 0.

$$P_A = P_A'(P/F, 8\%, 2)$$

The total present worth is determined by adding P_A and the initial payment P_0 in year 0.

$$\begin{aligned} P_T &= P_0 + P_A \\ &= 5000 + 500(P/A, 8\%, 6)(P/F, 8\%, 2) \\ &= 5000 + 500(4.6229)(0.8573) \\ &= \$6981.60 \end{aligned}$$

2.6. Using Spreadsheets for Equivalency Computation

The easiest single-cell spreadsheet functions to apply to find *P*, *F,* or *A* require that the cash flows exactly fit the function format. The functions apply the correct sign to the answer that would be on the cash flow diagram. That is, if cash flows are deposits (minus), the answer will have a plus sign. In order to retain the sign of the inputs, enter a minus sign prior to the function. Here is a summary and examples at 5% per year.

- **Present worth *P*:** Use the *PV* function = *(i%, n, A, F)* if *A* is exactly the same for each of *n* years; *F* can be present or not. For example, if *A* = $3000 per year deposit for *n* =

10 years, the function = *PV (5%, 10, -3000)* will display *P = $23,165*. This is the same as using the *P/A* factor to find *P = 3000(P/A,5%,10) = 3000(7.7217) = $23,165.*

- **Future worth *F*:** Use the FV function = *FV (i%, n, A, P)* if *A* is exactly the same for each of *n* years; *P* can be present or not. For example, if *A* = $3000 per year deposit for *n* = 10 years, the function = *FV (5%, 10, -3000)* will display *F* = $37,734. This is the same as using the *F/A* factor to find *F = 3000(F/A, 5%, 10) = 3000(12.5779) = $37,734.*
- **Annual amount *A*:** Use the *PMT* function = *PMT(i%, n, P, F)* when there is no *A* present, and either *P* or *F* or both are present. For example, for *P*= $-3000 deposit now and *F* = $5000 returned *n = 10* years hence, the function =*-PMT (5%, 10, -3000, 5000)* will display *A*= $9. This is the same as using the *A/P* and *A/F* factors to find the equivalent net *A*= $9 per year between the deposit now and return 10 years later.

  $$A = -3000\ (A/P,5\%,10) + 5000\ (A/F,5\%,10) = -389 + 398 = \$9$$

- **Number of periods *n*:** Use the *NPER* function = *NPER(i%, A, P, F)* if *A* is exactly the same for each of *n* years; either *P* or *F* can be omitted, but not both. For example, for *P* = $-25,000 deposit now and *A* = $3000 per year return, the function = *NPER(5%,3000, -25000)* will display *n* = 11.05 years to recover *P* at 5% per year. This is the same as using trial and error to find *n* in the relation *0 = -25,000 + 3,000 (P/A,5%, n)*.

When cash flows vary in amount or timing, it is usually necessary to enter them on a spreadsheet, including all zero amounts, and utilize other functions for P, F, or A values. All spreadsheet functions allow another function to be embedded in them, thus reducing the time necessary to get final answers. Example (2-8) illustrates these functions and the embedding capability.

Example (2-8)

Carol just entered college and her grandparents have offered her one of two gifts. They promised to give her $25,000 toward a new car if she graduates in 4 years. Alternatively, if she takes 5 years to graduate, they offered her $5000 each year starting after her second year is complete and an extra $5000 when she graduates. Draw the cash flow diagrams first. Then, use *i = 8%* per year to show Carol how to use spreadsheet functions and her financial calculator TVM functions to determine the following for *each gift* offered by her grandparents.

1) Present worth P now
2) Future worth F five years from now
3) Equivalent annual amount A over a total of 5 years
4) Number of years it would take Carol to have $25,000 in hand for the new

car if she were able to save $5000 each year starting next year.

Solution

Spreadsheet: The two cash flow series, labeled Gift A (lump sum) and Gift B (spread out), are in Figure (2-20). The spreadsheet in Figure (2-21a) lists the cash flows (don't forget to enter the $0 cash flows so the NPV function can be used), and answers to each part using the PV, NPV, FV, or PMT functions as explained below. In some cases, there are alternative ways to obtain the answer. Figure (2-21b) shows the function formulas with some comments. Remember that the PV, FV, and PMT functions will return an answer with the opposite sign from that of the cash flow entries. The same sign is maintained by entering a minus before the function name.

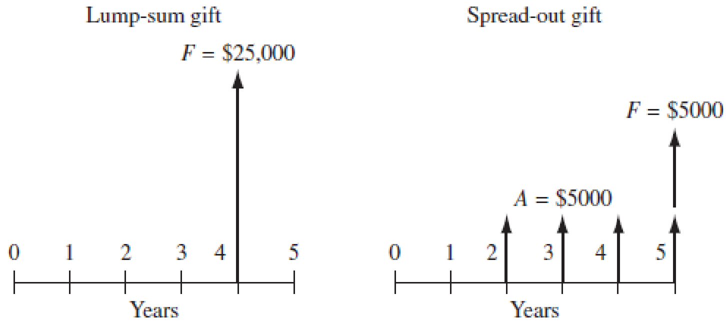


Figure (2–20): Cash flows for Carol's gift from her grandparents, Example (2-8).

1) Rows 12 and 13: There are two ways to find *P*; either the PV or NPV function. NPV requires that the zeros be entered. (For Gift A, omitting zeros in years 1, 2, and 3 will give the incorrect answer of *P*= $23,148, because NPV assumes the $25,000 occurs in year 1 and discounts it only one year at 8%.) The single-cell PV is hard to use for Gift B since cash flows do not start until year 2; using NPV is easier.
2) Rows 16 and 17: There are two ways to use the FV function to find *F* at the end of year 5. To develop FV correctly for Gift B in a single cell without listing cash flows, add the extra $5000 in year 5 separate from the FV for the four *A* = $5000 values. Alternatively, cell D17 embeds the NPV function for the *P* value into the FV function. This is a very convenient way to combine functions.
3) Rows 20 and 21: There are two ways to use the PMT function to find *A* for 5 years; find *P* separately and use a cell reference or embed the NPV function into the PMT to find *A* in one operation.
4) Row 24: Finding the years to accumulate $25,000 by depositing $5000 each year using the NPER function is independent of either plan. The entry = *NPER (8%, -5000,25000)* results in 4.3719 years. This can be confirmed by calculating *5000(F/A,8%,4.3719) = 5000(5.0000) = $25,000* (The 4.37 years is about the time it will take Carol to finish college. Of course, this assumes she can actually save $5000 a year while working on the degree.)

Calculator: Table (2-3) shows the format and completed calculator function for each gift, followed by the numerical answer below it. Minus signs on final answers have been changed to plus as needed to reflect the same sense as that in the spreadsheet solution. When calculating the values for Gift B, the functions can be performed separately, as shown, or embedded in the same way as the spreadsheet functions are embedded in Figure (2-21). In all cases, the answers are identical for the spreadsheet and calculator solutions.

	A	B	C	D
1			Cash flow, $	
2		Year	Gift A	Gift B
3		0		
4		1	0	0
5		2	0	5,000
6		3	0	5,000
7	Be sure to enter each zero cash flow	4	25,000	5,000
8		5	0	10,000
11		Function applied		
12	a. Present worth now	PV (single cell)	$18,376	
13		NPV		$18,737
16	b. Future worth; year 5	FV (single cell)	$27,000	$27,531
17		FV with embedded NPV		$27,531
20	c. Annual worth; years 1–5	PMT (reference P)	$4,602	$4,693
21		PMT with embedded NPV	$4,602	$4,693
24	d. Years to $25,000	NPER for both gifts	4.37	4.37

(*a*)

	A	B	C	D	E
1			Cash flow, $		
2		Year	Gift A	Gift B	
3		0			
4		1	0	0	
5		2	0	5000	
6		3	0	5000	
7		4	25000	5000	
8		5	0	10000	
11		Function applied			
12	a. Present worth now	PV with n = 4	= -PV(8%,4,,25000)		
13		NPV	= NPV(8%,C4:C8)	= NPV(8%,D4:D8)	
16	b. Future worth; year 5	FV	= -FV(8%,1,,25000)	= -FV(8%,4,5000) + 5000	
17		FV with embedded NPV		= -FV(8%,5,,NPV(8%,D4:D8))	
20	c. Annual worth; years 1–5	PMT (reference P)	= -PMT(8%,5,C12)	= -PMT(8%,5,D13)	
21		PMT with embedded NPV	= -PMT(8%,5,NPV(8%,C4:C8))	= -PMT(8%,5,NPV(8%,D4:D8))	
24	d. Years to $25,000	NPER (same for both)	= NPER(8%,-5000,,25000)	= NPER(8%,-5000,,25000)	

Function recognizes extra $5000 in year 5 as F value; no cash flow listing needed

Embedded NPV finds P in year 0

Functions reference P determined using PV or NPV function

(*b*)

Figure (2–21): (a) Use of several spreadsheet functions to find P, F, A, and n values, and (b) format of functions to obtain values, Example (2-8).

Table (2–3): Solution Using Calculator TVM Functions, Example (2-8)

Year	Cash flow, $	
	Gift A	Gift B
0		
1	0	0
2	0	5,000
3	0	5,000
4	25,000	5,000
5	0	5,000 + 5,000
	Functions applied	
a. Present worth now	PV(i,n,A,F) PV(8,4,0,25000) **$18,376**	FV(i,n,A,P) + 5,000 FV(8,4,5000,0) + 5,000 **$27,531** PV($i,n,A,F$) PV(8,5,0,27531) **$18,737**
b. Future worth, year 5	FV(i,n,A,P) FV(8,1,0,25000) **$27,000**	FV(i,n,A,P) + 5,000 FV(8,4,5000,0) + 5,000 **$27,531**
c. Annual worth, years 1–5	PMT(i,n,P,F) PMT(8,5,0,27000) **$4,602**	PMT(i,n,P,F) PMT(8,5,0,27531) **$4,693**
d. Years to $25,000	n(i,A,P,F) n(8,−5000,0,25000) **4.37**	n(i,A,P,F) n(8,−5000,0,25000) **4.37**

Problems

1- Find the correct numerical value for the following factors from the interest tables:
a) (F/P,10%,20)
b) (A/F,4%,8)
c) (P/A,8%,20)
d) (A/P,20%,28)
e) (F/A,30%,15)

2- What is the present worth of $30,000 in year 8 at an interest rate of 10% per year?

3- The Moller Skycar M400 is a flying car known as a personal air vehicle (PAV). The cost is $995,000, and a $100,000 deposit holds one of the first 100 vehicles. Assume a buyer pays the $885,000 balance 3 years after making the $100,000 deposit. At an interest rate of 10% per year, determine the effective total cost of the PAV in year 3 using:
a) Tabulated factors,
b) A single-cell spreadsheet function.

4- How much will be in an investment account 12 year from now if you deposit $3000 now and $5000 four years from now and the account earns interest at a rate of 10% per year? Use:
a) Tabulated factor values,
b) TVM functions on a financial calculator,
c) Built-in functions on a spreadsheet.

5- Ametek Technical & Industrial Products (ATIP) manufactures brushless blowers for boilers, foodservice equipment, and fuel cells. The company borrowed $17,000,000 for a plant expansion and repaid the loan in eight annual payments of $2,737,680, with the first payment made one year after the company received the money. What interest rate did ATIP pay? Develop the answer using:
a) Tabulated factor values,
b) A financial calculator,
c) Spreadsheet functions.

6- How many years will it take for money to increase to three times the initial amount at an interest rate of 10% per year?

7- Acceleron is planning future expansion with a new facility in Indianapolis. The company will make the move when its real estate sinking fund has a total value of $1.2 million. If the fund currently has $400,000 and the company adds $50,000 per year, how many years will it take for the account to reach the desired value? The fund earns interest at a rate of 10% per year.

8- Silastic-LC-50 is a liquid silicon rubber designed to provide high clarity, superior mechanical properties, and short cycle time for high speed manufacturers. One high-volume manufacturer used it to achieve smooth release from molds. The company's projected growth would result in silicon costs of $26,000 next year and costs increasing by $2000 per year through year 5. The interest rate is 10% per year.
a) What is the present worth of these costs using tabulated factors?
b) How is this problem solved using a spreadsheet? Using a financial calculator?

9- For the cash flows shown, determine the value of G that makes the present worth in year 0 equal to $14,000. The interest rate is 10% per year.

Year	0	1	2	3	4
Cash flow, $ per year	—	8000	8000-G	8000-2G	8000-3G

10- For the cash flow series shown, determine the future worth in year 5 at an interest rate of 10% per year.

Year	1	2	3	4	5
Cash flow, $	300,000	275,000	250,000	225,000	200,000

11- Attenuated Total Reflectance (ATR) is a method for looking at the surfaces of materials that are too opaque or too thick for standard transmission methods. A manufacturer of precision plastic parts estimates that ATR spectroscopy can save the company $8000 per year by reducing returns of out-of-spec parts. What is the future worth of the savings if they start now and extend through year 4? Use i = 10% per year.

12- For the cash flows shown, calculate the future worth in year 8 at i = 10% per year.

Year	0	1	2	3	4	5	6
Cash flow, $	100	100	100	200	200	200	200

13- Encon Systems, Inc. sales revenues for a product line introduced 7 years ago is shown. Use tabulated factors, a calculator or a spreadsheet to calculate the equivalent annual worth over the 7 years using an interest rate of 10% per year.

Year	**Revenue, $**
0	4,000,000
1	4,000,000
2	4,000,000
3	4,000,000
4	5,000,000
5	5,000,000
6	5,000,000
7	5,000,000

Chapter 3

Nominal and Effective Interest Rates

Chapter 3
Nominal and Effective Interest Rates

General Objective:

Trainee will be able to understand the Nominal and effective interest rates

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

1. Difference Between Nominal and Effective Interest Rates.
2. Calculating the Effective Interest Rate.
3. Formulation Equivalence Calculations Involving Only Single Amount Factors.
4. Equivalence Calculations Involving Series with PP ≥ CP and with PP < CP.

Introduction

In all engineering economy relations developed thus far, the interest rate has been a constant, annual value. For a substantial percentage of the projects evaluated by engineers in practice, the interest rate is compounded more frequently than once a year; frequencies such as semiannual, quarterly, and monthly are common. In fact, weekly, daily, and even continuous compounding may be experienced in some project evaluations. Also, in our own personal lives, many of the financial considerations we make—loans of all types (home mortgages, credit cards, automobiles, boats), checking and savings accounts, investments, stock option plans, etc.—have interest rates compounded for a time period shorter than 1 year. This requires the introduction of two new terms—nominal and effective interest rates. This chapter explains how to understand and use nominal and effective interest rates in engineering practice and in daily life situations.

3.1. Nominal and Effective Interest Rate Statements

In Chapter 1, we learned that the primary difference between simple interest and compound interest is that compound interest includes interest on the interest earned in the previous period, while simple interest does not. Here we discuss *nominal and effective interest rates,* which have the same basic relationship. The difference here is that the concepts of nominal and effective are used when interest is compounded more than once each year. For example, if an interest rate is expressed as 1% per month, the terms *nominal* and *effective* interest rates must be considered. Every nominal interest rate *must* be converted into an effective rate before it can be used in formulas, factor tables, calculator, or spreadsheet functions because they are all derived using effective rates.

The term *APR (Annual Percentage Rate)* is often stated as the annual interest rate for credit cards, loans, and house mortgages. This is the same as the *nominal rate*. An APR of 15% is the same as nominal 15% per year or a nominal 1.25% per month.

Also, the term *APY (Annual Percentage Yield)* is a commonly stated annual rate of return for investments, certificates of deposit, and savings accounts. This is the same as an *effective rate*. As we will discover, the nominal rate never exceeds the effective rate, and similarly APR < APY.

Before discussing the conversion from nominal to effective rates, it is important to *identify* a stated rate as either nominal or effective. There are three general ways of expressing interest rates as shown by the three groups of statements in Table (3-1). The three statements in the top third of the table show that an interest rate can be stated over some designated time period without specifying the compounding period. Such interest rates are assumed to be effective rates with the *compounding period (CP)* the same as that of the stated interest rate.

For the interest statements presented in the middle of Table (3-1), three conditions prevail: (1) The compounding period is identified, (2) this compounding period is shorter than the time period over which the interest is stated, and (3) the interest rate is designated neither as nominal nor as effective. In such cases, the interest rate is assumed to be *nominal* and the compounding period is equal to that which is stated. (We learn how to get effective interest rates from these in the next section).

For the third group of interest-rate statements in Table (3-1), the word *effective* precedes or follows the specified interest rate, and the compounding period is also given. These interest rates are obviously effective rates over the respective time periods stated.

The importance of being able to recognize whether a given interest rate is nominal or effective cannot be overstated with respect to the reader's understanding of the remainder of the material in this chapter and indeed the rest of the book. Table (3-2) contains a listing of several interest statements (column 1) along with their interpretations (columns 2 and 3).

Table (3-1): Various Interest Statements and Their Interpretations

(1) **Interest Rate Statement**	**(2)** **Interpretation**	**(3)** **Comment**
i = 12% per year	i = *effective* 12% per year	When no compounding period is given, interest rate is an effective rate, with compounding period assumed to be equal to stated time period.
i = 1% per month	i = *effective* 1% per month compounded monthly	
$i = 3^{1/2}$% per quarter	i = *effective* $3^{1/2}$% per quarter compounded quarterly	
i = 8% per year compounded monthly	i = *nominal* 8% per year compounded monthly	When compounding period is given without stating whether the interest rate is nominal or effective, it is assumed to be nominal. Compounding period is as stated.
i = 4% per quarter compounded monthly	i = *nominal* 4% per quarter compounded monthly	
i = 14% per year compounded compounded semiannually	i = *nominal* 14% per year compounded compounded semiannually	
i = APY of 10% per year compounded monthly	i = *effective* 10% per year compounded monthly	If interest rate is stated as an effective or APY rate, then it is an effective rate. If compounding period is not given, compounding period is assumed to coincide with stated time period.
i = effective 6% per quarter	i = *effective* 6% per quarter compounded quarterly	
i = effective 1% per month compounded daily	i = *effective* 1% per month compounded daily	

Table (3-2): Specific Examples of Interest Statements and Interpretations

(1) **Interest Rate Statement**	**(2)** **Nominal or Effective Interest**	**(3)** **Compounding Period**
15% per year compounded monthly	Nominal	Monthly
15% per year	Effective	Yearly
Effective 15% per year compounded monthly	Effective	Monthly
20% per year compounded quarterly	Nominal	Quarterly
Nominal 2% per month compounded weekly	Nominal	Weekly
2% per month	Effective	Monthly
2% per month compounded monthly	Effective	Monthly
Effective 6% per quarter	Effective	Quarterly
Effective 2% per month compounded daily	Effective	Daily
1% per week compounded continuously	Nominal	Continuously

3.2. Effective Interest Rate Formulation

Understanding effective interest rates requires a definition of a nominal interest rate *r* as the interest rate per period times the number of periods. In equation form,

$$r = \text{interest rate per period} \times \text{number of periods} \qquad (3-1)$$

A nominal interest rate can be found for any time period that is longer than the compounding period. For example, an interest rate of 1.5% per month can be expressed as a *nominal* 4.5% per quarter (1.5% per period ×3 periods), 9% per semiannual period, 18% per year, or 36% per 2 years. Nominal interest rates obviously neglect compounding.

The equation for converting a nominal interest rate into an effective interest rate is

$$i\ per\ period = (1 + r/m)^{m} - 1 \qquad (3-2)$$

where *i* is the *effective* interest rate for a certain period, say six months, *r* is the *nominal* interest rate for that *period* (six months here), and *m* is the number of times interest is *compounded in that same period* (six months in this case). The term *m* is often called the *compounding frequency*. As was true for nominal interest rates, effective interest rates can be calculated for any time period longer than the compounding period of a given interest rate. The next example illustrates the use of Equations (3-1) and (3-2).

Example (3-1)

1) A Visa credit card issued through Frost Bank carries an interest rate of 1% per month on the unpaid balance. Calculate the effective rate per semiannual and annual periods.
2) If the card's interest rate is stated as 3.5% per quarter, find the effective semiannual and annual rates.

Solution

1) The compounding period is monthly. For the effective interest rate per semiannual period, the r in Equation (3-1) must be the nominal rate per 6 months.

$$\begin{aligned} r &= 1\%\ per\ month \times 6\ months\ per\ semiannual\ period \\ &= 6\%\ \ per\ semiannual\ period \end{aligned}$$

The *m* in Equation (3-2) is equal to 6, since the frequency with which interest is compounded is 6 times in 6 months. The effective semiannual rate is

$$\begin{aligned} i\ per\ 6\ months &= (1 + 0.06/6)^{6} - 1 \\ &= 0.0615 = (6.15\%) \end{aligned}$$

For the effective annual rate, r = 12% per year and m = 12. By Equation (3-2),

$$\begin{aligned} Effective\ i\ per\ year &= (1 + 0.12/12)^{12} - 1 \\ &= 0.1268 = (12.68\%) \end{aligned}$$

2) For an interest rate of 3.5% per quarter, the compounding period is a quarter. In a semiannual period, m = 2 and r = 7%.

$$\begin{aligned} i\ per\ 6\ months &= (1 + 0.07/2)^{2} - 1 \\ &= 0.0712 = (7.12\%) \end{aligned}$$

The effective interest rate per year is determined using *r* = 14% and m = 4.

$$\begin{aligned} i\ per\ 6\ year &= (1 + 0.14/4)^{4} - 1 \\ &= 0.1475 = (14.75\%) \end{aligned}$$

If we allow compounding to occur more and more frequently, the compounding period becomes shorter and shorter. Then *m*, the number of compounding periods per payment period, increases. This situation occurs in businesses that have a very large number of cash flows every day, so it is correct to consider interest as compounded continuously. As *m* approaches infinity, the effective interest rate in Equation (3-2) reduces to

$$i = e^{r} - 1 \qquad (3-3)$$

Equation (3-3) is used to compute the *effective continuous interest rate.* The time periods on *i* and *r* must be the same. As an illustration, if the nominal annual *r* = 15% *per year,* the effective continuous rate *per year* is

$$i\% = e^{0.15} - 1 = 16.183\%$$

For national and international chains—retailers, banks, etc.—and corporations that move thousands of items in and out of inventory each day, the flow of cash is essentially continuous. *Continuous cash flow* is a realistic model for the analyses performed by engineers and others in these organizations. The equivalence computations reduce to the use of integrals rather than summations. The topics of financial engineering analysis and continuous cash flows, coupled with continuous interest rates, are beyond the scope of this text; consult more advanced texts for formulas and procedures.

Example (3-2)

1) For an interest rate of 18% per year compounded continuously, calculate the effective monthly and annual interest rates.
2) An investor requires an effective return of at least 15%. What is the minimum annual nominal rate that is acceptable for continuous compounding?

Table (3-3): Effective Annual Interest Rates for Selected Nominal Rates

Nominal Rate r%	Semiannually (m = 2)	Quarterly (m = 4)	Monthly (m = 12)	Weekly (m = 252)	Daily (m = 365)	Continuously (m = ∞; e^r-1)
2	2.010	2.015	2.018	2.020	2.020	2.020
4	4 4.040	4.060	4.074	4.079	4.081	4.081
5	5.063	5.095	5.116	5.124	5.126	5.127
6	6.090	6.136	6.168	6.180	6.180	6.184
8	8.160	8.243	8.300	8.322	8.328	8.329
10	10.250	10.381	10.471	10.506	10.516	10.517
12	12.360	12.551	12.683	12.734	12.745	12.750
15	15.563	15.865	16.076	16.158	16.177	16.183
18	18.810	19.252	19.562	19.684	19.714	19.722

Solution

1) The nominal monthly rate is r = 18%/12 = 1.5%,or 0.015 per month. By Equation (3-3), the effective monthly rate is

$$i\%\ per\ month = e^{r} - 1 = e^{0.015} - 1 = 1.511\%$$

Similarly, the effective annual rate using r = 0.18 per year is

$$i\%\ per\ year = e^{r} - 1 = e^{0.18} - 1 = 19.72\%$$

2) Solve Equation (3-3) for r by taking the natural logarithm.

$$e^{r} - 1 = 0.15$$

$$e^{r} = 1.15$$

$$ln e^{r} = ln 1.15$$

$$r\% = 13.976\%$$

Therefore, a nominal rate of 13.976% per year compounded continuously will generate an effective 15% per year return.

Comment: The general formula to find the nominal rate, given the effective continuous rate i, is $r = \ln(1 + i)$

3.3. Reconciling Compounding Periods and Payment Periods

Now that the concepts of nominal and effective interest rates are introduced, in addition to considering the compounding period (which is also known as the interest period), it is necessary to consider the frequency of the payments or receipts within the cash-flow time interval. For simplicity, the frequency of the payments or receipts is known as the *payment period (PP)*. It is important to distinguish between the compounding period (CP) and the payment period because in many instances the two do not coincide. For example, if a company deposited money each month into an account that pays a nominal interest rate of 6% per year compounded semiannually, the payment period is 1 month while the compounding period is 6 months as shown in Figure (3-1). Similarly, if a person deposits money once each year into a savings account that compounds interest quarterly, the payment period is 1 year, while the compounding period is 3 months. Hereafter, for problems that involve either uniform-series or gradient cash-flow amounts, it will be necessary to determine the relationship between the compounding period and the payment period as a first step in the solution of the problem.

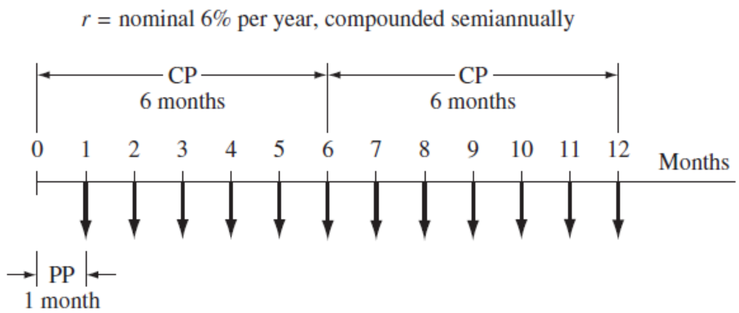


Figure (3-1): Cashflow diagram for a monthly payment period (PP) and semiannual compounding period (CP).

The next three sections describe procedures for determining the correct *i* and *n* values for use in formulas, and factor tables, as well as calculator and spreadsheet functions. In general, there are three steps:

1- Compare the lengths of PP and CP.
2- Identify the cash-flow series as involving only single amounts (*P* and *F*) or series amounts (*A*, *G*, or *g*).
3- Select the proper *i* and *n* values.

3.4. Formulation Equivalence Calculations Involving Only Single Amount Factors

There are many correct combinations of *i* and *n* that can be used when only single amount factors (*F*/*P* and *P*/*F*) are involved. This is because there are only two requirements: (1) An effective rate must be used for *i*, and (2) the time unit on *n* must be the same as that on *i*. In standard factor notation, the single-payment equations can be generalized.

$$P = F(P/F, effective\ i\ per\ period, number\ of\ periods) \qquad (3-4)$$

$$F = P(F/P, effective\ i\ per\ period, number\ of\ periods) \qquad (3-5)$$

Thus, for a nominal interest rate of 12% per year compounded monthly, any of the i and corresponding n values shown in Table (3-4) could be used (as well as many others not shown) in the factors. For example, if an effective quarterly interest rate is used for i, that is, $(1.01)^3$-1 = 3.03%, then the n time unit is 4 quarters in a year.

Table (3-4): Various i and n Values for Single- Amount Equations Using r =12% per Year, Compounded Monthly

Effective Interest Rate, i	Units for n
1% per month	Months
3.03% per quarter	Quarters
6.15% per 6 months	Semiannual periods
12.68% per year	Years
26.97% per 2 years	2-year periods

Alternatively, it is always correct to determine the effective *i* per payment period using Equation (3-2) and to use standard factor equations to calculate *P*, *F*, or *A*.

Example (3-3)

Sherry expects to deposit $1000 now, $3000 4 years from now, and $1500 6 years from now and earn at a rate of 12% per year compounded semiannually through a company-sponsored savings plan. What amount can she withdraw 10 years from now?

Solution

Only single-amount P and F values are involved (Figure (3-2)). Since only effective rates can be present in the factors, use an effective rate of 6% per semiannual compounding period and semiannual payment periods. The future worth is calculated using Equation (3-5).

$$F = 1000(F/P, 6\%, 20) + 3000\ (F/P, 6\%, 12) + 1500\ (F/P, 6\%, 8) = \$11.634$$

An alternative solution strategy is to find the effective annual rate by Equation (3-2) and express *n* in years as determined from the problem statement.

$$Effective\ i\ per\ year = (1 + 0.12/2)^{\ 2} - 1 = 0.1236\ (12.36\%)$$

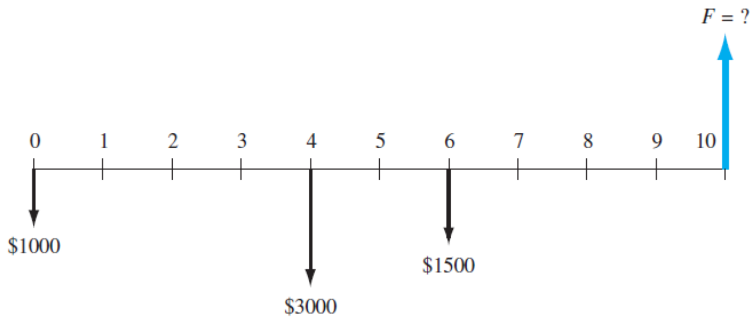


Figure (3-2): Cashflow diagram, Example (3-3)

3.5. Equivalence Calculations Involving Series with PP ≥ CP

When the cash flow of the problem dictates the use of one or more of the uniform series or gradient factors, the relationship between the compounding period, CP, and payment period, PP, must be determined. The relationship will be one of the following three cases:

Type 1. Payment period equals compounding period, *PP* = *CP*.

Type 2. Payment period is longer than compounding period, *PP* > *CP*.

Type 3. Payment period is shorter than compounding period, *PP* < *CP*.

The procedure for the first two types is the same. Type 3 problems are discussed in the following section. When *PP* = *CP or PP* > *CP*, the following procedure *always* applies:

Step 1. Count the number of payments and use that number as *n*. For example, if payments are made quarterly for 5 years, *n* is 20.

Step 2. Find the *effective* interest rate over the *same time period* as *n* in step 1. For example, if *n* is expressed in quarters, then the effective interest rate per quarter *must* be used.

Use these values for *n* and *i* (and only these!) in the factors, functions, or formulas. To illustrate, Table (3-5) shows the correct standard notation for sample cash-flow sequences and interest rates. Note in column 4 that *n* is always equal to the number of payments and *i* is an effective rate expressed over the same time period as *n*.

Table (3-5): Examples of n and i Values Where PP = CP or PP > CP

(1) Cash-flow Sequence	(2) Interest Rate	(3) What to Find; What is Given	(4) Standard Notation
\$500 semiannually for 5 years	8% per year compounded semiannually	Find *P*; given *A*	*P*= 500(*P*/*A*,4%,10)
\$75 monthly for 3 years	12% per year compounded monthly	Find *F*; given *A*	*F* = 75 (*F*/*A*,1%,36)
\$180 quarterly for 15 years	5% per quarter	Find *F*; given *A*	*F* = 180 (*F*/*A*,5%,60)
\$25 per month increase for 4 years	1% per month	Find *P*; given *G*	*P* = 25 (*P*/*G*,1%,48)
\$5000 per quarter for 6 years	1% per month	Find *P*; given *A*	*P* = 5000 (*P*/*A*,3.03%,24)

Example (3-4)

For the past 7 years, a quality manager has paid \$500 every 6 months for the software maintenance contract on a laser-based measuring instrument. What is the equivalent amount after the last payment, if these funds are taken from a pool that has been returning 10% per year compounded quarterly?

Solution

The cash flow diagram is shown in Figure (3-3). The payment period (6 months) is longer than the compounding period (quarter); that is, PP > CP. Applying the guideline, determine an effective semiannual interest rate. Use Equation (3-2) or Table (3-3) with *r* = 0.05 per 6-month period and *m = 2* quarters per semiannual period.

$$Effective\ i\ per\ 6\ months = (1 - 0.05/2)^{\ 2} - 1$$
$$= 5.063\%$$

The value *i* = 5.063% is reasonable, since the effective rate should be slightly higher than the nominal rate of 5% per 6-month period. The number of semiannual periods is *n* = 2(7) = 14 The future worth is

$$F = A(\mathrm{F/A}, 5.063\%, 14)$$
$$= 500\ (19.6845) = \$9842$$

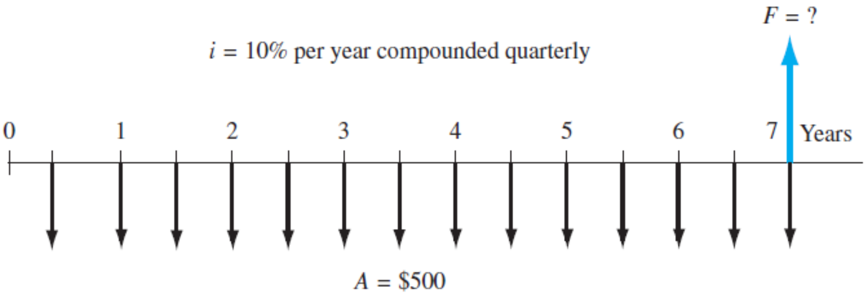


Figure (3-3): Diagram of semiannual payments used to determine F, Example (3-4)

3.6. Equivalence Calculations Involving Series with PP < CP

If a person deposits money each *month* into a savings account where interest is compounded *quarterly,* do the so-called *inter-period deposits* earn interest? The usual answer is no. However, if a monthly payment on a $10 million, quarterly compounded bank loan were made early by a large corporation, the corporate financial officer would likely insist that the bank reduce the amount of interest due, based on early payment. These two are examples of PP < CP, type 3 cash flows. The timing of cash flow transactions between compounding points introduces the question of how *inter-period compounding* is handled. Fundamentally, there are two policies: inter-period cash flows earn *no interest,* or they earn *compound interest*. The only condition considered here is the first one (no interest), because many real world transactions fall into this category.

For a no-inter-period-interest policy, deposits (negative cash flows) are all regarded as *deposited at the end of the compounding period,* and withdrawals are all regarded as *withdrawn at the beginning.* As an illustration, when interest is compounded quarterly, all monthly deposits are moved to the end of the quarter, and all withdrawals are moved to the beginning (no interest is paid for the entire quarter). This procedure can significantly alter the distribution of cash flows before the effective quarterly rate is applied to find *P*, *F*, or *A*. This effectively forces the cash flows into a PP = CP situation, as discussed in Section 5.

Example (3-5)

Rob is the on-site coordinating engineer for Alcoa Aluminum, where an under- renovation mine has new ore refining equipment being installed by a local contractor. Rob developed the cash flow diagram in Figure (3-4a) in $1000 units from the project perspective. Included are payments to the contractor he has authorized for the current year and approved advances from Alcoa's home office. He knows that the interest rate on equipment "field projects" such as this is 12% per year compounded quarterly, and that Alcoa does not bother with inter-period compounding of interest. Will Rob's project finances be in the "red" or the "black" at the end of the year? By how much?

Solution

With no inter-period interest considered, Figure (3-4a) reflects the moved cash flows. The future worth after four quarters requires an *F* at an effective rate per quarter such that PP = CP = 1 quarter, therefore, the effective i = 12%/4 = 3%. Figure (3-4b) shows all negative cash flows (payments to contractor) moved to the end of the respective quarter, and all positive cash flows (receipts from home office) moved to the beginning of the respective quarter. Calculate the *F* value at 3%.

$$F = 1000[-150(F/P,3\%,4) - 200(F/P,3\%,3) + (-175 + 90)(F/P,3\%,2) + 165(F/P,3\%,1) - 50] = \$-357{,}592$$

Rob can conclude that the on-site project finances will be in the red about $357,600 by the end of the year.

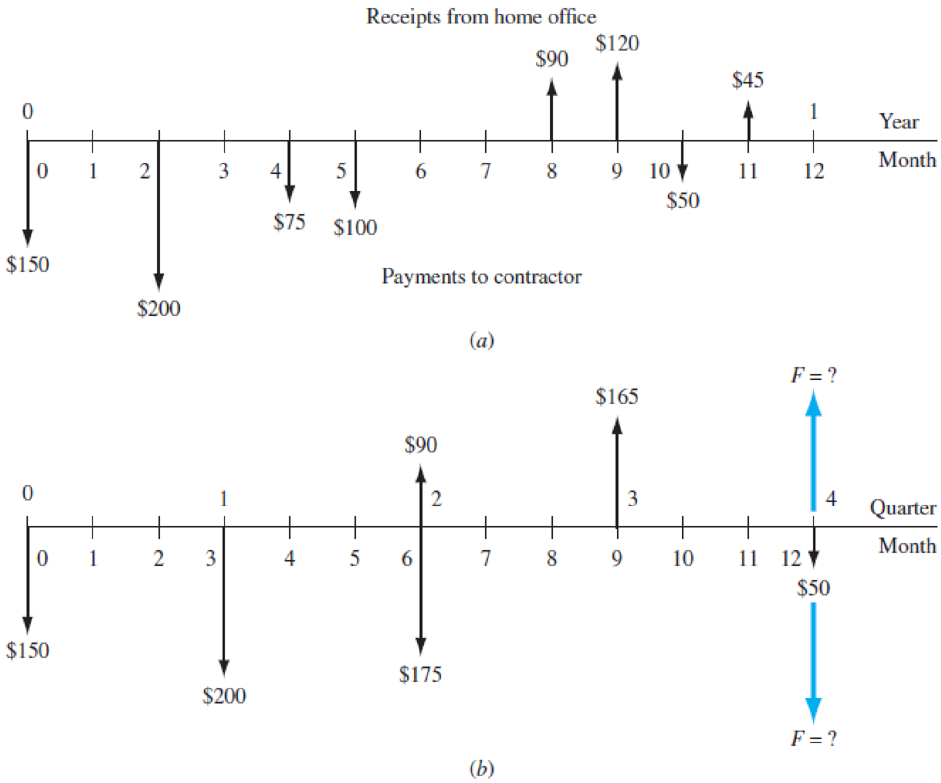


Figure (3-4): (a) Actual and (b) moved cash flows (in $1000) for quarterly compounding periods using no inter-period interest,, Example (3-5)

Problems

1- For an interest rate of 2% per quarter, determine the nominal interest rate per:
a) Semiannual
b) Year
c) 2 years

2- Identify each of the following interest rate statements as either nominal or effective.
a) 4% per year
b) 6% per year compounded annually
c) 10% per quarter
d) 8% per year compounded monthly
e) 1% per month
f) 1% per month compounded monthly
g) 0.1% per day compounded hourly
h) Effective 1.5% per month compounded weekly
i) 12% per year compounded semiannually
j) 1% per month compounded continuously

3- The TerraMax truck currently being manufactured and field tested by Oshkosh Truck Co. is a driverless truck intended for military use. Such a truck would free personnel for non-driving tasks such as reading maps, scanning for roadside bombs, or scouting for the enemy. If such trucks would result in reduced injuries to military personnel amounting to $15 million three years from now, determine the present worth of these benefits at an interest rate of 10% per year compounded semiannually.

4- For the cash flows shown, determine the future worth in year 5 at an interest rate of 10% per year compounded continuously. Solve using :
a) Tabulated factors,
b) A financial calculator.

Year	1	2	3	4	5
Cash flow, $	300,000	0	250,000	0	200,000

5- Improvements at a Harley-Davidson Plant are estimated to be $7.8 million. Construction is expected to take three years. What is the future worth of the project in year 3 at an interest rate of 6% per year compounded quarterly, assuming the funds are allocated
a) Completely at time 0,
b) Equally at the end of each year?

6- Using tabulated factors for the cash flow series shown, calculate the future worth at the end of quarter 6 using i = 8% per year compounded quarterly. What are the calculator functions?

Quarter	0	1	2	3	4	5	6
Cash flow, $	100	100	300	300	300	300	300

7- For the cash flows shown, determine the future worth in year 5 at an interest rate of 1% per month.

Year	1	2	3	4	5
Cash flow, $	300,000	275,000	250,000	225,000	200,000

8- In an effort to save money for early retirement, an environmental engineer plans to deposit \$1200 per month starting one month from now, into a self-directed investment account that pays 8% per year compounded semiannually. How much will be in the account at the end of 25 years?

9- You plan to invest \$1000 per month in a stock that pays a dividend at a rate of 4% per year compounded quarterly. How much will the stock be worth at the end of 9 years if there is no inter-period compounding?

10- Western Refining purchased a model MTVS peristaltic pump for injecting antiscalant at its Nano-filtration water conditioning plant. The cost of the pump was \$1200. If the chemical cost is \$11 per day, determine the equivalent cost per month at an interest rate of 1% per month. Assume 30 days per month and a 4-year pump life.

Chapter 4

Present Worth Analysis

Chapter 4
Present Worth Analysis

General Objective:

Trainee will be able to understand the Present Worth Analysis

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

1. Present Worth Analysis of Equal-Life Alternatives.
2. Present Worth Analysis of Different-Life Alternatives.
3. Capitalized Cost Analysis.
4. Evaluation of Independent Projects.
5. Using Spreadsheets for Present Worth Analysis.

Introduction

Alternatives are developed from project proposals to accomplish a stated purpose. The logic of alternative formulation and evaluation is depicted in Figure (4-1). Some projects are economically and technologically viable, and others are not. Once the viable projects are defined, it is possible to formulate the alternatives. Alternatives are one of two types: mutually exclusive or independent. Each type is evaluated differently.

- **Mutually exclusive (ME).** *Only one of the viable projects can be selected.* Each viable project *is* an alternative. If no alternative is economically justifiable, do nothing (DN) is the default selection.
- **Independent.** *More than one viable project may be selected* for investment. (There may be dependent projects requiring a particular project to be selected before another, and/or contingent projects where one project may be substituted for another.)

An alternative or project is comprised of estimates for the first cost, expected life, salvage value, and annual costs. Salvage value is the best estimate of an anticipated future market or trade-in value at the end of the expected life. Salvage may be estimated as a percentage of the first cost or an actual monetary amount; salvage is often estimated as nil or zero. Annual costs are commonly termed annual operating costs (AOC) or maintenance and operating (M&O) costs. They may be uniform over the entire life, increase or decrease each year as a percentage or arithmetic gradient series, or vary over time according to some other expected pattern.

A mutually exclusive alternative selection is the most common type in engineering practice. It takes place, for example, when an engineer must select the one best diesel-powered engine from several competing models. Mutually exclusive alternatives are, therefore, the same as the viable projects; each one is evaluated, and the one best alternative is chosen. Mutually exclusive alternatives *compete with one another* in the evaluation. All the analysis techniques compare mutually exclusive alternatives. Present worth is discussed in the remainder of this chapter.

The *do-nothing (DN)* option is usually understood to be an alternative when the evaluation is performed. If it is absolutely required that one of the defined alternatives be selected, do nothing is not considered an option. (This may occur when a mandated function must be installed for safety, legal, or other purposes.) Selection of the DN alternative means that the current approach is maintained; no new costs, revenues, or savings are generated.

Independent projects are usually designed to accomplish different purposes, thus the possibility of selecting any number of the projects. These projects (or bundles of projects) do not compete with one another; each project is evaluated separately, and the *comparison is* with the MARR.

Finally, it is important to classify an *alternative's* cash flows as revenue-based or cost-based. All alternatives evaluated in one study must be of the same type.

- **Revenue.** *Each alternative generates cost and revenue cash flow estimates, and possibly savings,* which are treated like revenues. Revenues may be different for each alternative. These alternatives usually involve new systems, products, and services that require capital investment to generate revenues and/or savings. Purchasing new equipment to increase productivity and sales is a revenue alternative.
- **Cost.** *Each alternative has only cost cash flow estimates.* Revenues are assumed to be equal for all alternatives. These may be public sector (government) initiatives, or legally mandated or safety improvements. Cost alternatives are compared to each

other; do-nothing is not an option when selecting from mutually exclusive cost alternatives.

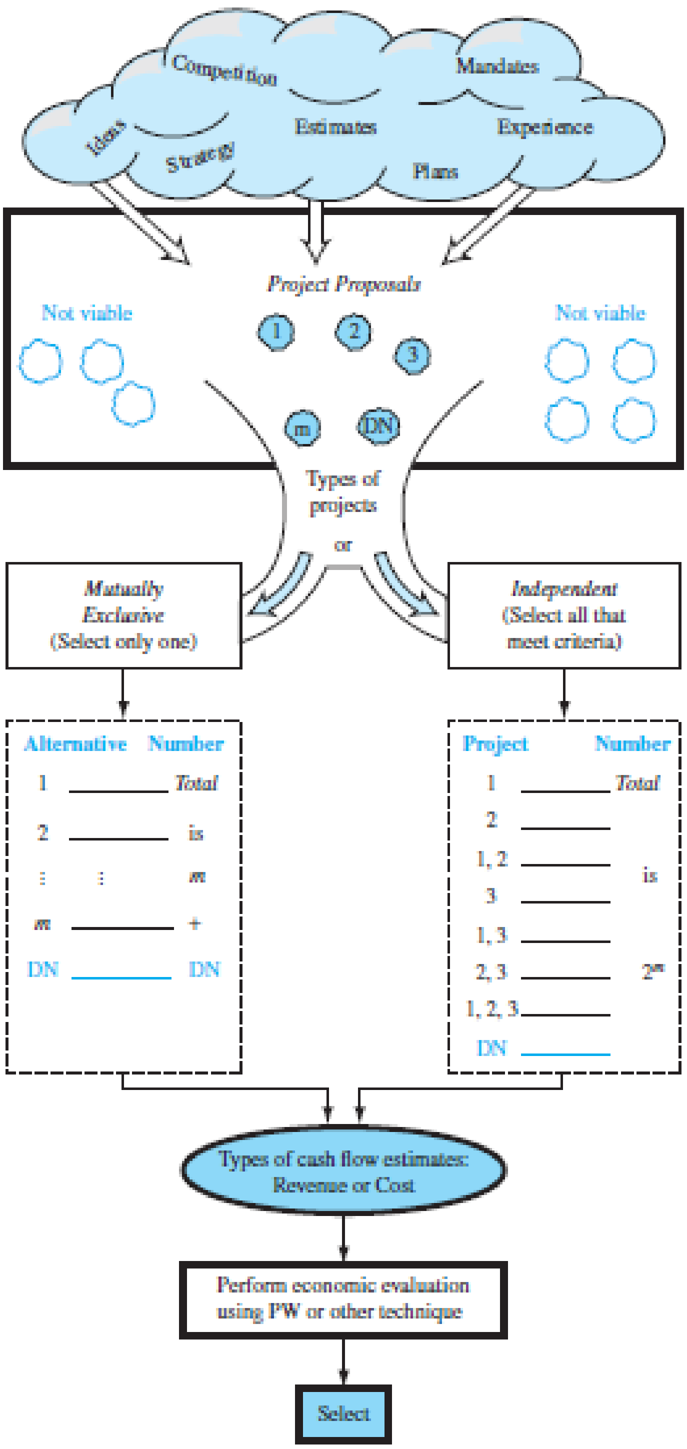


Figure (4-1): Logical progression from proposals to alternatives to selection.

4.1. Present Worth Analysis of Equal-Life Alternatives.

The PW comparison of alternatives with equal lives is straightforward. The present worth P is renamed PW of the alternative. The present worth method is quite popular in industry because all future costs and revenues are transformed to **equivalent monetary units NOW;**

that is, all future cash flows are converted (discounted) to present amounts (e.g., dollars) at a specific rate of return, which is the MARR. This makes it very simple to determine which alternative has the best economic advantage. The required conditions and evaluation procedure are as follows:

If the alternatives have the same capacities for the same time period (life), the equal-service requirement is met. Calculate the PW value at the stated MARR for each alternative.

For **mutually exclusive (ME)** alternatives, whether they are revenue or cost alternatives, the following guidelines are applied to justify a single project or to select one from several alternatives.

One alternative: If PW ≥ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the PW that is numerically largest, that is, less negative or more positive. This indicates a lower PW of cost for cost alternatives or a larger PW of net cash flows for revenue alternatives.

Note that the guideline to select one alternative with the lowest cost or highest revenue uses the criterion of **numerically largest.** This is not the absolute value of the PW amount, because the sign matters. The selections below correctly apply the guideline for two alternatives A and B.

PW_A	PW_B	Selected Alternative
$-2300	$-1500	B
-500	+1000	B
+ 2500	+2000	A
+4800	-400	A

For **independent** projects, each PW is considered separately, that is, compared with the DN project, which always has PW = 0. The selection guideline is as follows:

One or more independent projects: Select all projects with PW ≥ 0 at the MARR.

The independent projects must have positive and negative cash flows to obtain a PW value that can exceed zero; that is, they must be revenue projects.

Example (4-1)

A university lab is a research contractor to NASA for in-space fuel cell systems that are hydrogen and methanol based. During lab research, three equal-service machines need to be evaluated economically. Perform the present worth analysis with the costs shown below. The MARR is 10% per year.

	Electric-Powered	Gas-Powered	Solar-Powered
First cost, $	-4500	-3500	-6000
Annual operating cost (AOC), $/year	-900	-700	-50
Salvage value *S*, $	200	350	100
Life, years	8	8	8

Solution

These are cost alternatives. The salvage values are considered a “negative” cost, so a + sign precedes them. (If it costs money to dispose of an asset, the estimated disposal cost has a -

sign.) The PW of each machine is calculated at i = 10% for n = 8 years. Use subscripts E , G , and S .

$$PW_E = -4500 - 900(P/A, 10\%, 8) + 200(P/F, 10\%, 8) = \$-9280$$

$$PW_G = -3500 - 700(P/A, 10\%, 8) + 3500(P/F, 10\%, 8) = \$-7071$$

$$PW_S = -6000 - 50(P/A, 10\%, 8) + 100(P/F, 10\%, 8) = \$-6220$$

The solar-powered machine is selected since the PW of its costs is the lowest; it has the numerically largest PW value.

Example (4-2)

As discussed in the introduction to this chapter, ultrapure water (UPW) is an expensive commodity for the semiconductor industry. With the options of seawater or groundwater sources, it is a good idea to determine if one system is more economical than the other. Use a MARR of 12% per year and the present worth method to select one of the systems.

Solution

An important first calculation is the cost of UPW per year. The general relation and estimated costs for the two options are as follows:

$$\boldsymbol{UPW\ cost\ relation}: = \frac{\$}{Year} = \left(\frac{cost\ in\ \$}{1000\ gallons}\right)\left(\frac{gallons}{minute}\right)\left(\frac{minutes}{hour}\right)\left(\frac{hours}{day}\right)\left(\frac{days}{year}\right)$$

$$Seawater: (4/1000)(1500)(60)(16)(250) = \$1.44\ M\ per\ year$$

$$Groundwater: (5/1000)(1500)(60)(16)(250) = \$1.80\ M\ per\ year$$

Calculate the PW at i = 12% per year and select the option with the lower cost (larger PW value). In \$1 million units:

$$\boldsymbol{PW\ relation}: PW = first\ cost - PW\ of\ AOC - PW\ of\ UPW + PW\ of\ salvage\ value$$

$$PW_s = -20 - 0.5(P/A, 12\%, 10) - 1.44(P/A, 12\%, 10) + 0.05(20)(P/F, 12\%, 10)$$

$$= -20 - 0.5(5.6502) - 1.44(5.6502) + 1(0.3220) = \$-30.64$$

$$PW_G = -22 - 0.3(P/A, 12\%, 10) - 1.80(P/A, 12\%, 10) + 0.10(22)(P/F, 12\%, 10)$$

$$= -22 - 0.2(5.6502) - 1.80(5.6502) + 2.2(0.3220) = \$-33.16$$

Based on this present worth analysis, the seawater option is cheaper by \$2.52 M.

4.2. Present Worth Analysis of Different-Life Alternatives

When the present worth method is used to compare mutually exclusive alternatives that have different lives, the equal-service requirement must be met.

> ***The PW of the alternatives must be compared over the same number of years and must end at the same time to satisfy the equal-service requirement.***

This is necessary, since the present worth comparison involves calculating the equivalent PW of all future cash flows for each alternative. A fair comparison requires that PW values represent cash flows associated with equal service. For cost alternatives, failure to compare equal service will always favor the shorter-lived mutually exclusive alternative, even if it is not the more economical choice, because fewer periods of costs are involved. The equal-service requirement is satisfied by using either of two approaches:

LCM: *Compare the PW of alternatives over a period of time equal to the* ***least common multiple (LCM)*** *of their estimated lives.*

Study period: *Compare the PW of alternatives using a* ***specified study period of n years.***

This approach does not necessarily consider the useful life of an alternative. The study period is also called the planning horizon.

For either approach, calculate the PW at the MARR and use the same selection guideline as that for equal-life alternatives. The LCM approach makes the cash flow estimates extend to the same period, as required. For example, lives of 3 and 4 years are compared over a 12-year period. The first cost of an alternative is reinvested at the beginning of each life cycle, and the estimated salvage value is accounted for at the end of each life cycle when calculating the PW values over the LCM period. Additionally, the LCM approach requires that some assumptions be made about subsequent life cycles.

The assumptions when using the LCM approach are that
1. ***The service provided will be needed over the entire LCM years or more.***
2. ***The selected alternative can be repeated over each life cycle of the LCM in exactly the same manner.***
3. ***Cash flow estimates are the same for each life cycle.***

A study period analysis is necessary if the first assumption about the length of time the alternatives are needed cannot be made. For the study period approach, a time horizon is chosen over which the economic analysis is conducted, and only those cash flows which occur during that time period are considered relevant to the analysis. All cash flows occurring beyond the study period are ignored. An estimated market value at the end of the study period must be made. The time horizon chosen might be relatively short, especially when short-term business goals are very important. The study period approach is often used in replacement analysis. It is also useful when the LCM of alternatives yields an unrealistic evaluation period, for example, 5 and 9 years.

Example (4-3)

National Homebuilders, Inc., plans to purchase new cut-and-finish equipment. Two manufacturers offered the estimates below.

	Vendor A	Vendor B
First cost, $	-15,000	-18,000
Annual M&O cost (AOC), $/year	-3,500	-3,100
Salvage value, $	1,000	2,000
Life, years	6	9

1- Determine which vendor should be selected on the basis of a present worth comparison, if the MARR is 15% per year.

2- National Homebuilders has a standard practice of evaluating all options over a 5-year period. If a study period of 5 years is used and the salvage values are not expected to change, which vendor should be selected?

Solution

1- Since the equipment has different lives, compare them over the LCM of 18 years. For life cycles after the first, the first cost is repeated in year 0 of each new cycle, which is the last year of the previous cycle. These are years 6 and 12 for vendor A and year 9 for B. The cash flow diagram is shown in Figure (4–2). Calculate PW at 15% over 18 years.

$$PW_A = -15.000 - 15.000(P/F, 15\%, 6) + 1000(P/F, 15\%, 6)$$
$$-15.000(P/F, 15\%, 12) + 1000(P/F, 15\%, 12) + 1000(P/F, 15\%, 18)$$
$$- 3.500(P/A, 15\%, 18) = \$ - 45.036$$

$$PW_B = -18.000 - 18.000(P/F, 15\%, 9) + 2000(P/F, 15\%, 9)$$
$$+2000(P/F, 15\%, 18) - 3100(P/A, 15\%, 8) = \$ - 41.384$$

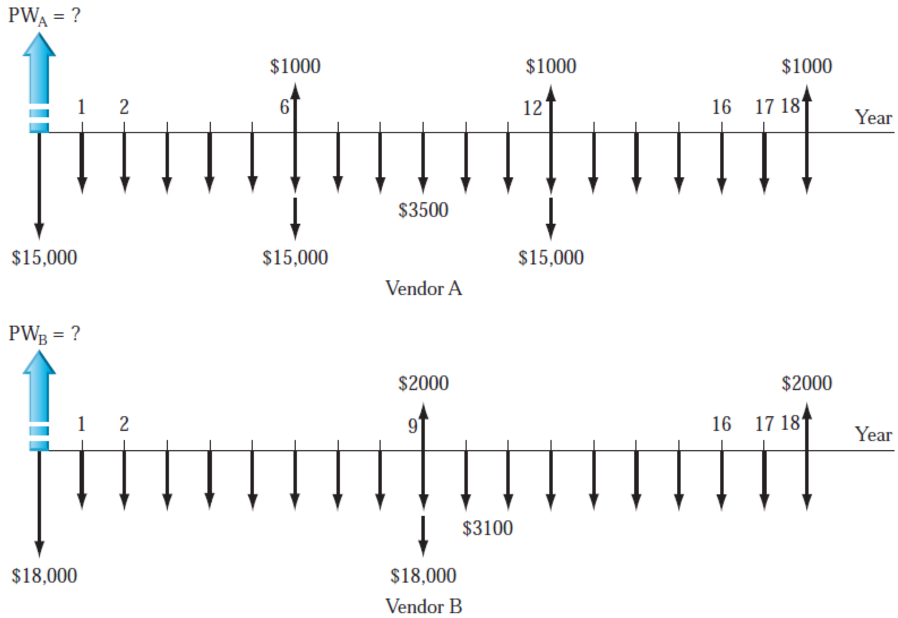


Figure (4-2): Cash flow diagram for different-life alternatives, Example (4-3).

Vendor B is selected, since it costs less in PW terms; that is, the PW_B value is numerically larger than PW_A.

2- For a 5-year study period, no cycle repeats are necessary. The PW analysis is

$$PW_A = -15.000 - 3500(P/A, 15\%, 5) + 1000(P/F, 15\%, 5) = \$ - 26.236$$
$$PW_B = -18.000 - 3100(P/A, 15\%, 5) + 2000(P/F, 15\%, 5) = \$ - 27.397$$

Vendor A is now selected based on its smaller PW value. This means that the shortened study period of 5 years has caused a switch in the economic decision. In situations such as this, the standard practice of using a fixed study period should be carefully examined to ensure that the appropriate approach, that is, LCM or fixed study period, is used to satisfy the equal-service requirement.

4.3. Capitalized Cost Analysis

Many public sector projects such as bridges, dams, highways and toll roads, railroads, and hydroelectric and other power generation facilities have very long expected useful lives. A **perpetual or infinite life** is the effective planning horizon. Permanent endowments for charitable organizations and universities also have perpetual lives. The economic worth of these types of projects or endowments is evaluated using the present worth of the cash flows.

> ***Capitalized Cost (CC)*** *is the present worth of a project that has a very long life (more than, say, 35 or 40 years) or when the planning horizon is considered very long or infinite.*

The formula to calculate CC is derived from the PW relation *P*/*A* (*P*/*A* , *i* %, *n*), where $n = \infty$ time periods. Take the equation for *P* using the *P*/*A* factor and divide the numerator and denominator by $(1 + i)^n$ to obtain

$$P = A\left[\frac{1 - \frac{1}{(1+i)^n}}{1}\right]$$

As *n* approaches ∞, the bracketed term becomes $1/i$. We replace the symbols *P* and PW with CC as a reminder that this is a capitalized cost equivalence. Since the *A* value can also be termed AW for annual worth, the capitalized cost formula is simply

$$CC = \frac{A}{i} or\ CC = \frac{AW}{i} \quad (4-1)$$

Solving for A or AW, the amount of new money that is generated each year by a capitalization of an amount CC is

$$AW = CC(i) \quad (4-2)$$

This is the same as the calculation *A/P(i)* for an infinite number of time periods. Equation (5-2) can be explained by considering the time value of money. If $20,000 is invested now (this is the capitalization) at 10% per year, the maximum amount of money that can be withdrawn at the end of every year for eternity is $2000, which is the interest accumulated each year. This leaves the original $20,000 to earn interest so that another $2000 will be accumulated the next year. The cash flows (costs, revenues, and savings) in a capitalized cost calculation are usually of two types: recurring, also called periodic, and nonrecurring. An annual operating cost of $50,000 and a rework cost estimated at $40,000 every 12 years are examples of recurring cash flows. Examples of nonrecurring cash flows are the initial investment amount in year 0 and one-time cash flow estimates at future times, for example, $500,000 in fees 2 years hence.

The procedure to determine the CC for an infinite sequence of cash flows is as follows:

1. Draw a cash flow diagram showing all nonrecurring (one-time) cash flows and at least two cycles of all recurring (periodic) cash flows.
2. Find the present worth of all nonrecurring amounts. This is their CC value.
3. Find the *A* value through *one life cycle* of all recurring amounts. (This is the same value in all succeeding life cycles) Add this to all other uniform amounts (*A*) occurring in years 1 through infinity. The result is the total equivalent uniform annual worth (AW).
4. Divide the AW obtained in step 3 by the interest rate *i* to obtain a CC value. This is an application of Equation (4-1).
5. Add the CC values obtained in steps 2 and 4.

Drawing the cash flow diagram (step 1) is more important in CC calculations than elsewhere, because it helps separate nonrecurring and recurring amounts. In step 5 the present worth's of all component cash flows have been obtained; the total capitalized cost is simply their sum.

Example (4-4)

The Haverty County Transportation Authority (HCTA) has just installed new software to charge and track toll fees. The director wants to know the total equivalent cost of all future costs incurred to purchase the software system. If the new system will be used for the indefinite future, find the equivalent cost:

a- now, a CC value,

b- for each year hereafter, an AW value.

The system has an installed cost of \$150,000 and an additional cost of \$50,000 after 10 years. The annual software maintenance contract cost is \$5000 for the first 4 years and \$8000 thereafter. In addition, there is expected to be a recurring major upgrade cost of \$15,000 every 13 years. Assume that i = 5% per year for county funds.

Solution

a- The five-step procedure to find CC now is applied.

1. Draw a cash flow diagram for two cycles Figure (4–3).
2. Find the present worth of the nonrecurring costs of \$150,000 now and \$50,000 in year 10 at i = 5%. Label this CC_1

$$CC_1 = -150{,}000 - 50{,}000(P/F, 5\%, 10) = \$-180{,}695$$

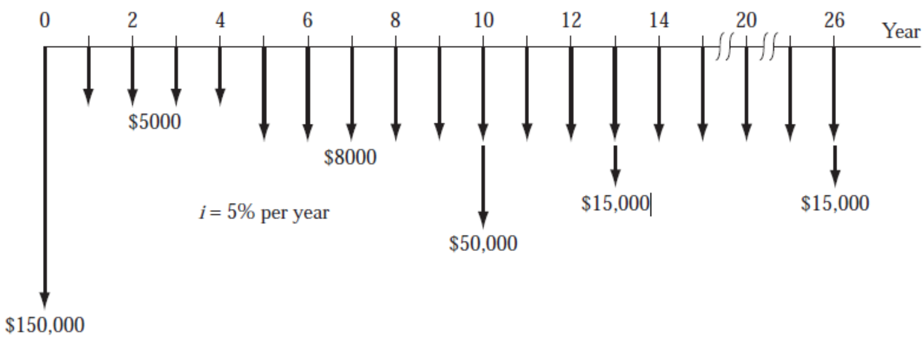


Figure (4-3): Cash flows for two cycles of recurring costs and all nonrecurring amounts, Example (4-4).

3. **And 4** .Convert the \$15,000 recurring cost to an A value over the first cycle of 13 years, and find the capitalized cost CC_2 at 5% per year using Equation (4-1).

$$A = -15{,}000(A/F, 5\%, 13) = \$-847$$

$$CC_2 = -780/0.05 = \$-16{,}940$$

There are several ways to convert the annual software maintenance cost series to *A* and CC values. A straightforward method is to, first, consider the \$−5000 an *A* series with a capitalized cost of

$$CC_3 = -5000/0.05 = \$-100{,}000$$

Second, convert the step-up maintenance cost series of \$−3000 to a capitalized cost CC_4 in year 4, and find the present worth in year 0. (Refer to Figure (4–4) for cash flow timings.)

$$CC_4 = (-3000/0.05)(P/F, 5\%, 4) = \$-49{,}362$$

5. The total capitalized cost CC_T for Haverty County Transportation Authority is the sum of the four component CC values.

$$CC_T = -180{,}695 - 16{,}940 - 100{,}000 - 49{,}362 = \$-346{,}997$$

b- Equation (4-2) determines the AW value forever.

$$AW = P_i = CC_T(i) = -346{,}997(0.05) = \$17{,}350$$

Correctly interpreted, this means Haverty County officials have committed the equivalent of $17,350 forever to operate and maintain the toll management software.

4.4. Evaluation of Independent Projects.

Consider a biomedical company that has a new genetics engineering product that it can market in three different countries (S, U, and R), including any combination of the three. The do-nothing (DN) alternative is also an option. All possible options are: S, U, R, SU, SR, UR, SUR, and DN. In general, for m independent projects, there are 2^m alternatives to evaluate. Selection from independent projects uses a fundamentally different approach from that for mutually exclusive (ME) alternatives. When selecting independent projects, each project's PW is calculated using the MARR. (In ME alternative evaluation, the projects compete with each other, and only one is selected.) The selection rule is quite simple for one or more *independent* projects:

Select all projects that have PW ≥ 0 at the MARR.

All projects must be developed to have revenue cash flows (not costs only) so that projects earning more than the MARR have positive PW values.

Unlike ME alternative evaluation, which assumes the need for the service over multiple life cycles, independent projects are considered one-time investments. This means the PW analysis is performed over the respective life of each project and the assumption is made that any leftover cash flows earn at the MARR when the project ends. As a result, the equal service requirement does not impose the use of a specified study period or the LCM method. The implied study period is that of the longest lived project.

There are two types of selection environments—unlimited and budget constrained.

- **Unlimited.** All projects that make or exceed the MARR are selected. Selection is made using the PW ≥ 0 guideline.
- **Budget constrained.** No more than a specified amount, *b*, of funds can be invested in all of the selected projects, and each project must make or exceed the MARR. Now the solution methodology is slightly more complex in that *bundles* of projects that do not exceed the investment limit *b* are the only ones evaluated using PW values. The procedure is:

  1. Determine all bundles that have total initial investments no more than *b*. (This limit usually applies in year 0 to get the project started).

  2. Find the PW value at the MARR for all projects contained in the bundles.

  3. Total the PW values for each bundle in (1).

  4. Select the bundle with the largest PW value.

4.5. Using Spreadsheets for Present Worth Analysis.

Spreadsheet- or calculator-based evaluation of equal-life, mutually exclusive alternatives can be performed using the single-cell PV function when the annual amount *A* is the same. The general format to determine the PW is

$$= P - PV(i, n, A, F) \quad (4-3)$$

It is important to pay attention to the sign placed on the PV function in order to get the correct answer for the alternative's PW value. The spreadsheet function returns the opposite sign of the A series. Therefore, to retain the negative sense of a cost series A, place a minus sign immediately in front of the PV function. This is illustrated in the next example.

Example (4-5)

Cesar, a petroleum engineer, has identified two equivalent diesel-powered generators to be purchased for an offshore platform. Use i = 12% per year to determine which is the more economic. Solve using both spreadsheet and calculator functions.

	Generator 1	Generator 2
P, $	-80,000	-120,000
S, $	15,000	40,000
n, years	3	3
AOC, $/year	-30,000	-8,000

Solution

Spreadsheet: Follow the format in Equation (4-3) in a single cell for each alternative. Figure (4-4) shows the details. Note the use of minus signs on P, the PV function, and AOC value. Generator 2 is selected with the smaller PW of costs (numerically larger value).
Calculator: The function and PW value for each alternative are:

Generator 1: -80000 - PV(12,3,-30000,15000) PW1 = $-141,378

Generator 2: -120000 - PV(12,3,-8000,40000) PW2 = $-110,743

As expected, the PW values and selection of Generator 2 are the same as the spreadsheet solution.

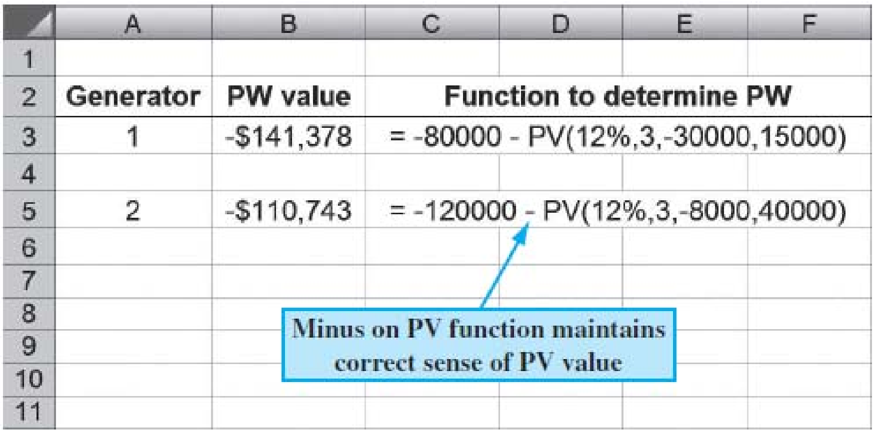


Figure (4-4): Equal-life alternatives evaluated using the PV function, Example (4-5).

When different-life alternatives are evaluated, using the LCM basis, it is necessary to input all the cash flows for the LCM of the lives to ensure an equal-service evaluation. Develop the NPV function to find PW. If cash flow is identified by CF, the general format is

$$= P + NPV(i\%, year\_1\_CF\_cell{:}\, last\_year\_CF\_cell) \quad (4-4)$$

It is very important that the *initial cost P not be included* in the cash flow series identified in the NPV function. Unlike the PV function, the NPV function returns the correct sign for the PW value.

Example (4-6)

Continuing with the previous example, once Cesar had selected generator 2 to purchase, he approached the manufacturer with the concerns that the first cost was too high and the expected life was too short. He was offered a lease arrangement for 6 years with a \$20,000 annual cost and an extra \$20,000 payment in the first and last years to cover installation and removal costs. Determine if generator 2 or the lease arrangement is better at 12% per year.

	A	B	C	D	E	F	G
1	Year	Generator 2	Lease				
2	0	-120,000	-40,000				
3	1	-8,000	-20,000				
4	2	-8,000	-20,000				
5	3	-88,000	-20,000				
6	4	-8,000	-20,000				
7	5	-8,000	-20,000				
8	6	32,000	-40,000				
9	PW value	-\$189,568	-\$132,361				
10							
11							
12							

Repurchase cash flow
= S - AOC - P
= 40,000 − 8,000 − 120,000

= −40000 + NPV(12%,C3:C8)

= −120000 + NPV(12%,B3:B8)

Figure (4-5):Different-life alternatives evaluated using the NPV function, Example (4-6).

Solution

Assuming that generator 2 can be repurchased 3 years hence and all estimates remain the same, PW evaluation over 6 years is correct. Figure 4.5 details the cash flows and NPV functions. The year 3 cash flow for generator 2 is S - AOC - P = \$ - 88,000. Note that the first costs are not included in the NPV function but are listed separately, as indicated in Equation (4-4). The lease option is the clear winner for the next 6 years.

When evaluating alternatives for which the annual cash flows do not form an A series, the individual amounts must be entered on the spreadsheet and Equation (4-4) is used to find PW values. Also, remember that any zero-cash-flow year must be entered as 0 to ensure that the NPV function correctly tracks the years.

Problems

1- State two conditions under which the do-nothing alternative is not an option.

2- What is the difference between mutually exclusive alternatives and independent projects?

3- When evaluating projects by the present worth method, how do you know which one(s) to select, if the

k) Projects are independent,

l) Alternatives are mutually exclusive?

4- A biomedical engineer with Johnston Implants just received estimates for replacement equipment to deliver online selected diagnostic results to doctors performing surgery who need immediate information on the patient's condition. The cost is $200,000, the annual maintenance contract costs $5000, and the useful life (technologically) is 5 years.

a) What is the alternative if this equipment is not selected? What other information is necessary to perform an economic evaluation of the two?

b) What type of cash flow series will these estimates form?

c) What additional information is needed to convert the cash flow estimates to the other type of series?

5- A company that manufactures magnetic membrane switches is investigating two production options that have the estimated cash flows shown ($1 million units). Which one should be selected on the basis of a present worth analysis at 10% per year?

	In-house	Contract
First cost, $	-30	0
Annual cost, $ per year	-5	-2
Annual income, $ per year	14	3.1
Salvage value, $	2	--
Life, years	5	5

6- What is meant by the term equal service?

7- Oil from a particular type of marine microalgae can be converted to biodiesel that can serve as an alternate transportable fuel for automobiles and trucks. If lined ponds are used to grow the algae, the construction cost will be $13 million and the maintenance & operating (M&O) cost will be $2.1 million per year. If long plastic tubes are used for growing the algae, the initial cost will be higher at $18 million, but less contamination will render the M&O cost lower at $0.41 million per year. At an interest rate of 10% per year and a 5-year project period, which system is better, ponds or tubes? Use a present worth analysis.

8- A wealthy businessman wants to start a permanent fund for supporting research directed toward sustainability. The donor plans to give equal amounts of money for each of the next 5 years, plus one now (i.e., six donations) so that $100,000 per year can be withdrawn each year forever, beginning in year 6. If the fund earns interest at a rate of 8% per year, how much money must be donated each time?

9- The cost of maintaining a certain permanent monument in Washington, DC occurs as periodic outlays of $1000 every year and $5000 every 4 years. Calculate the capitalized cost of the maintenance using an interest rate of 10% per year. Compare the alternatives shown on the basis of their capitalized costs using an interest rate of 10% per year.

	Alternative M	Alternative N
First cost, $	-150,000	-800,000
Annual cost, $ per year	-50,000	-12,000
Annual income, $ per year		
Salvage value, $	8,000	1,000,000
Life, years	5	∞

Chapter 5

Annual Worth Analysis

Chapter 5
Annual Worth Analysis

General Objective:

Trainee will be able to understand the Annual Worth Analysis

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

1. Advantages and Uses of Annual Worth Analysis.
2. AW Value Calculation.
3. Evaluating Alternatives Based on Annual Worth.
4. AW of a Permanent Investment.

Introduction

In this chapter, we add to our repertoire of alternative comparison tools. In Chapter 4 we learned the PW method. Here we learn the equivalent annual worth, or AW, method. AW analysis is commonly considered the more desirable of the two methods because the AW value is easy to calculate; the measure of worth—AW in monetary units per year—is understood by most individuals; and its assumptions are essentially identical to those of the PW method.

5.1. Advantages and Uses of Annual Worth Analysis

For many engineering economic studies, the AW method is the best to use, when compared to PW, FW, and rate of return (Chapters 6). Since the AW value is the equivalent uniform annual worth of all estimated receipts and disbursements during the life cycle of the project or alternative, AW is easy to understand by any individual acquainted with annual amounts, for example, dollars per year. The AW value, which has the same interpretation as A used thus far, is the economic equivalent of the PW and FW values at the MARR for n years. All three can be easily determined from each other by the relation

$$AW = PW(A/P, i, n) = FW(A/F, i, n) \qquad (5-1)$$

The *n* in the factors is the number of years for equal-service comparison. This is the LCM or the stated study period of the PW or FW analysis. When all cash flow estimates are converted to an AW value, this value applies for every year of the life cycle and for *each additional life cycle.*

The annual worth method offers a prime computational and interpretation advantage because the AW value needs to be calculated for only one life cycle. The AW value determined over one life cycle is the AW for all future life cycles. Therefore, it is not necessary to use the LCM of lives to satisfy the equal-service requirement.

As with the PW method, there are three fundamental assumptions of the AW method that should be understood. When alternatives being compared have different lives, the AW method makes the assumptions that

1. The services provided are needed for at least the LCM of the lives of the alternatives.
2. The selected alternative will be repeated for succeeding life cycles in exactly the same manner as for the first life cycle.
3. All cash flows will have the same estimated values in every life cycle.

In practice, no assumption is precisely correct. If, in a particular evaluation, the first two assumptions are not reasonable, a study period must be established for the analysis. Note that for assumption 1, the length of time may be the indefinite future (forever). In the third assumption, all cash flows are expected to change exactly with the inflation (or deflation) rate. If this is not a reasonable assumption, new cash flow estimates must be made for each life cycle, and again a study period must be used.

Example (5-1)

In Example (4-3), National Homebuilders, Inc. evaluated cut-and-finish equipment from vendor A (6-year life) and vendor B (9-year life). The PW analysis used the LCM of 18 years. Consider only the vendor A option now. The diagram in Figure (5–1) shows the cash flows for all three life cycles (first cost \$ -15,000; annual M&O costs \$ -3500; salvage value

$1000). Demonstrate the equivalence at i = 15% of PW over three life cycles and AW over one cycle. In Example (4-3), present worth for vendor A was calculated as PW = $ -45,036.

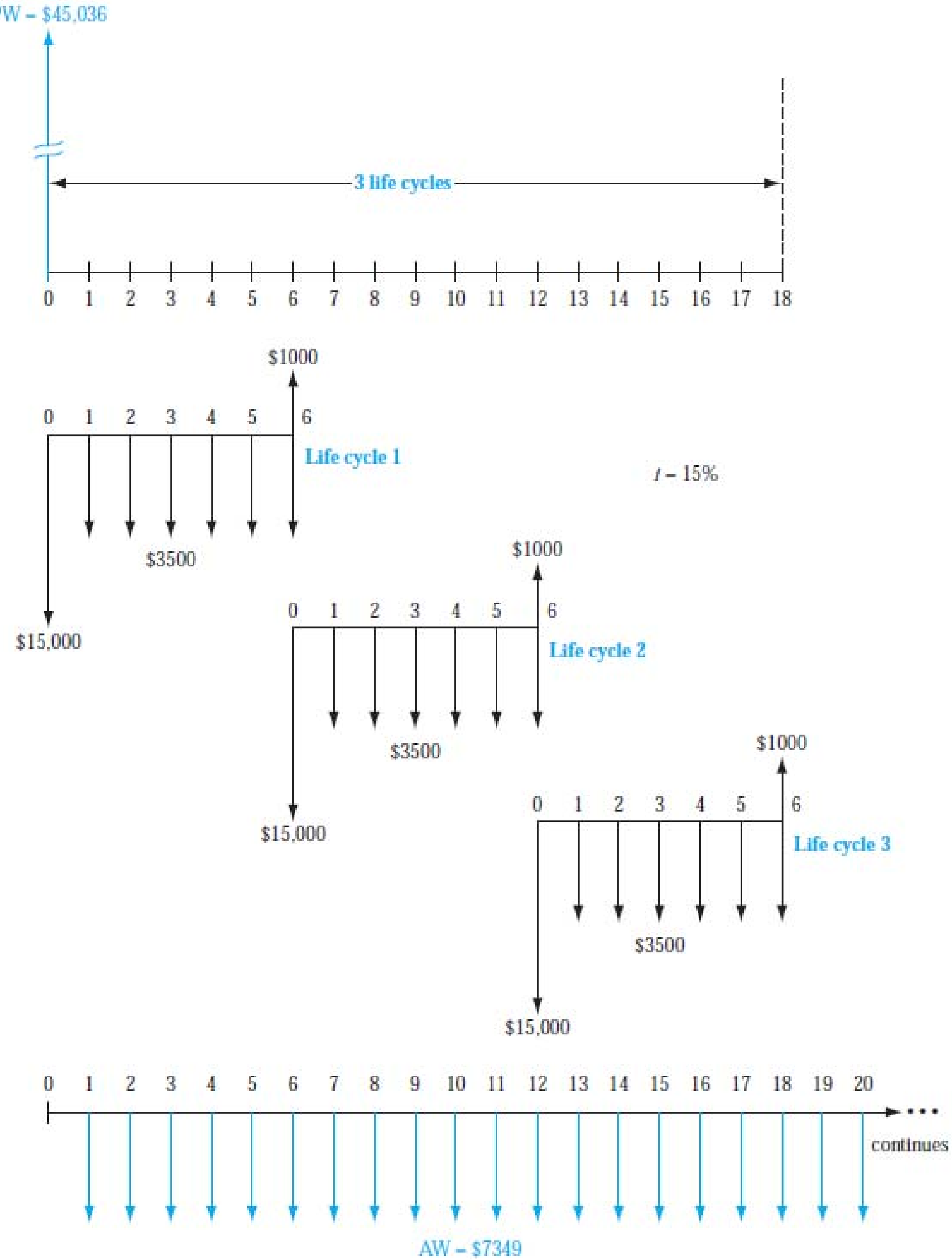


Figure (5-1): PW and AW values for three life cycles, Example (5-1).

Solution

Calculate the equivalent uniform annual worth value for all cash flows in the first life cycle.

$$AW = -15{,}000(A/P, 15\%, 6) + 1000(A/F, 15\%, 6) - 3500 = \$ -7349$$

When the same computation is performed on each succeeding life cycle, the AW value is $ -7349. Now Equation (5-1) is applied to the PW value for 18 years.

$$AW = -45{,}036(A/P, 15\%, 18) = \$ -7349$$

The one-life-cycle AW value and the AW value based on 18 years are equal.

5.2. AW Value Calculation

The AW value of an alternative is the addition of two distinct components: capital recovery (CR) of the initial investment and the equivalent A value of the annual operating costs (AOC).

$$AW = CR + A\ of\ AOC \qquad (5-2)$$

The recovery of an amount of capital *P* committed to an asset, plus the time value of the capital at a particular interest rate, is a fundamental principle of economic analysis. *Capital recovery is the equivalent annual cost of owning the asset plus the return on the initial investment.* The *A*/*P* factor is used to convert *P* to an equivalent annual cost. If there is some anticipated positive salvage value *S* at the end of the asset's useful life, its equivalent annual value is removed using the *A*/*F* factor. This action reduces the equivalent annual cost of owning the asset. Accordingly, CR is

$$CR = -P(A/P, i, n) + S(A/F, i, n) \qquad (5-3)$$

The annual amount (*A* of AOC) is determined from uniform recurring costs (and possibly receipts) and nonrecurring amounts. The *P*/*A* and *P*/*F* factors may be necessary to first obtain a present worth amount, then the *A*/*P* factor converts this amount to the *A* value in Equation (5-2).

Example (5-2)

Lockheed Martin is increasing its booster thrust power in order to win more satellite launch contracts from European companies interested in new global communications markets. A piece of earth-based tracking equipment is expected to require an investment of $13 million. Annual operating costs for the system are expected to start the first year and continue at $0.9 million per year. The useful life of the tracker is 8 years with a salvage value of $0.5 million. Calculate the AW value for the system if the corporate MARR is currently 12% per year.

Solution

The cash flows (Figure (5-2*a*)) for the tracker system must be converted to an equivalent AW cash flow sequence over 8 years (Figure (5.2*b*)). (All amounts are expressed in $1 million units.) The AOC is A = $ -0.9 per year, and the capital recovery is calculated by using Equation (5-3).

$$\begin{aligned} CR &= -13(A/P, 12\%, 8) + 0.5(A/F, 12\%, 8) \\ &= -13(0.2013) + 0.5(0.0813) \\ &= \$\ -2.576 \end{aligned}$$

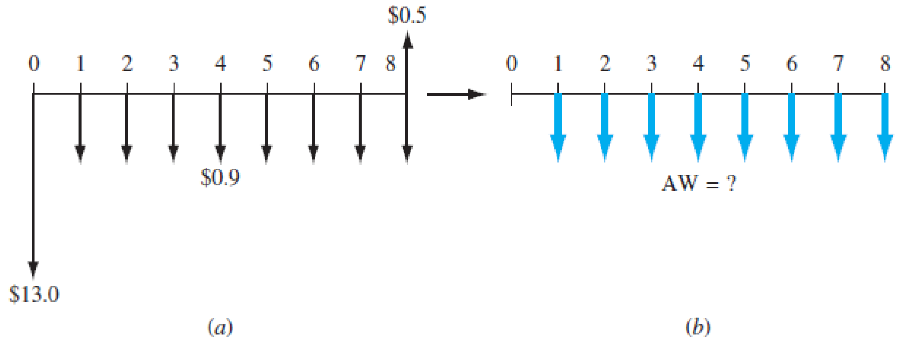


Figure (5-2): (a) Cash flow diagram for satellite tracker costs, and (b) conversion to an equivalent AW (in $1 million), Example (5-2).

The correct interpretation of this result is very important to Lockheed Martin. It means that each and every year for 8 years, the equivalent total revenue from the tracker must be at least $2,576,000 *just to recover the initial present worth investment plus the required return of 12% per year*. This does not include the AOC of $0.9 million each year. Total AW is found by Equation (5-2).

$$AW = -132.576 - 0.9 = \$ - 3.476 \, million \, per \, year$$

This is the AW for all future life cycles of 8 years, provided the costs rise at the same rate as inflation, and the same costs and services apply for each succeeding life cycle.

For solution by computer, use the PMT function to determine CR only in a single spreadsheet cell. The format is = PMT(*i*%,*n*,*P*,-*S*). As an illustration, the CR in Example (5-2) is displayed when = PMT(12%,8,13,-0.5) is entered.

The annual worth method is applicable in any situation where PW, FW, or Benefit/Cost analysis can be utilized. The AW method is especially useful in certain types of studies: asset replacement and retention studies to minimize overall annual costs, breakeven studies and make-or-buy decisions (all covered in later chapters), and all studies dealing with production or manufacturing where cost/unit is the focus.

5.3. Evaluating Alternatives Based on Annual Worth.

The annual worth method is typically the easiest of the evaluation techniques to perform, when the MARR is specified. The alternative selected has the lowest equivalent annual cost (cost alternatives), or highest equivalent income (revenue alternatives). The selection guidelines for the AW method are the same as for the PW method.

One alternative: AW ≥ 0, the alternative is financially viable.

Two or more alternatives: Choose the numerically largest AW value (lowest cost or highest income).

If a study period is used to compare two or more alternatives, the AW values are calculated using cash flow estimates over only the study period. For a study period shorter than the alternative's expected life, use an estimated market value for the salvage value.

Example (5-3)

PizzaRush, which is located in the general Los Angeles area, fares very well with its competition in offering fast delivery. Many students at the area universities and community colleges work part-time delivering orders made via the web at PizzaRush.com. The owner, a software engineering graduate of USC, plans to purchase and install five portable, in-car systems to increase delivery speed and accuracy. The systems provide a link between the web order-placement software and the in-car GPS system for satellite-generated directions to any address in the Los Angeles area. The expected result is faster, friendlier service to customers, and more income for PizzaRush.

Each system costs $4600, has a 5-year useful life, and may be salvaged for an estimated $300. Total operating cost for all systems is $650 for the first year, increasing by $50 per year thereafter. The MARR is 10% per year. Perform an annual worth evaluation that answers the following questions:

1- How much new annual revenue is necessary to recover only the initial investment at an MARR of 10% per year?
2- The owner conservatively estimates increased income of $5000 per year for all five systems. Is this project financially viable at the MARR? See cash flow diagram in Figure (5-3).
3- Based on the answer in part (2), determine how much new income PizzaRush must have to economically justify the project. Operating costs remain as estimated.

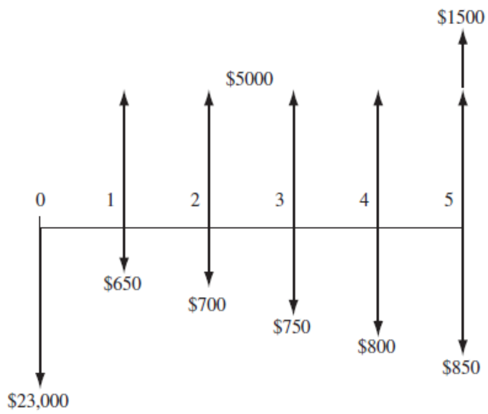


Figure (5-3): Cash flow diagram used to compute AW, Example (5-3).

Solution

1- The CR value will answer this question. Use Equation (5-3) at 10%.

$$CR = -5(4600)(A/P, 10\%, 5) + 5(300)(A/F, 10\%, 5)$$
$$= \$ -5822$$

2- The financial viability could be determined now without calculating the AW value, because the $5000 in new income is lower than the CR of $5822, which does not yet include the annual costs. So, the project is not economically justified. However, to complete the analysis, determine the total AW. The annual operating costs and incomes form an arithmetic gradient series with a base of $4350 in year 1, decreasing by $50 per year for 5 years. The AW relation is

$$AW = -\,capital\ recovery + \ A\ of\ net\ income$$
$$= -5822 + 4350 - 50(A/G, 10\%, 5) \qquad (5-4)$$
$$= \$ -1562$$

This shows conclusively that the alternative is not financially viable at MARR = 10%.

3- An equivalent of the projected \$5000 plus the AW amount are necessary to make the project economically justified at a 10% return. This is 5000 + 1562 = \$6562 per year in new revenue. At this point AW will equal zero based on Equation (5-4).

5.4. AW of a Permanent Investment.

The annual worth equivalent of a very long-lived project is the AW value of its capitalized cost (CC), discussed in Section 4 chapter 4. The AW value of the first cost, *P*, or present worth, PW, of the alternative uses the same relation as Equation (4-2).

$$AW = CC(i) = PW(i) \qquad (5-5)$$

Cash flows that occur at regular intervals are converted to AW values over one life cycle of their occurrence. All other nonregular cash flows are first converted to a *P* value and then multiplied by *i* to obtain the AW value over infinity.

Example (5-4)

If you receive an inheritance of \$10,000 today, how long do you have to invest it at 8% per year to be able to withdraw \$2000 every year forever? Assume the 8% per year is a return that you can depend on forever.

Solution

Cash flow is detailed in Figure (5-4). Solving Equation (5-5) for PW indicates that it is necessary to have \$25,000 accumulated at the time that the \$2000 annual withdrawals start.

$$PW = 2000/0.08 = \$\,25{,}000$$

$$Find\ n = 11.91\ years\ using\ the\ relation\ \$25{,}000 = 10.000(P/F, 8\%, n)$$

Comment: It is easy to use a spreadsheet to solve this problem. In any cell write the function = NPER(8%,,-10000,25000) to display the answer of 11.91 years. The financial calculator function *n*(8,0,-10000,25000) displays the same *n* value.

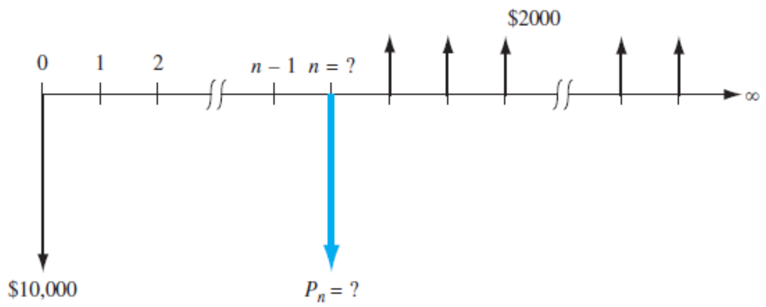


Figure (5-4): Diagram to determine n for a perpetual withdrawal, Example (5-4).

Problems

1- Heyden Motion Solutions ordered $7 million worth of seamless tubes for manufacturing their high performance and precision linear motion products. If their annual operating costs are $860,000 per year, how much annual revenue is required over a 3-year planning period to recover the initial investment and operating costs at the company's MARR of 15% per year?.

2- NRG Energy plans to construct a giant solar plant in Santa Teresa, NM to supply electricity to 30,000 southern NM and western TX homes. The plant will have 390,000 heliostats to concentrate sunlight onto 32 water towers to generate steam. NRG will spend $560 million in constructing the plant and $430,000 per year in operating it. If a salvage value of 20% of the initial cost is assumed, how much will the company have to make each year for 15 years in order to recover its investment at a MARR of 18% per year?

3- Two machines with the following cost estimates are under consideration for a dishwasher assembly process. Using an interest rate of 10% per year, determine which alternative should be selected on the basis of an annual worth analysis.

	Machine X	Machine Y
First cost, $	-300,000	-430,000
Annual operating cost, $ per year	-60,000	-40,000
Salvage value, $	70,000	95,000
Life, years	4	6

4- An engineer is considering two different liners for an evaporation pond that will receive salty concentrate from a brackish water desalting plant. A plastic liner will cost $0.90 per square foot and will have to be replaced in 20 years when precipitated solids have to be removed from the pond using heavy equipment. A rubberized elastomeric liner is tougher and, therefore, is expected to last 30 years, but it will cost $2.20 per square foot. The pond covers 110 acres (1 acre _ 43,560 square feet). Which liner is more cost effective on the basis of an annual worth analysis at an interest rate of 8% per year? Solve using:

a) tabulated factors,
b) a calculator.

5- Calculate the equivalent annual cost for years 1 through infinity of $1,000,000 now and $1,000,000 three years from now at an interest rate of 10% per year.

6- Compare the alternatives below using the annual worth method at an interest rate of 10% per year. Use

a) tabulated factors,
b) calculator functions.

	A	B
First cost, $	-60,000	-380,000
Annual operating cost, $ per year	-30,000	-5000
Salvage value, $	10,000	25,000
Life, years	3	∞

Chapter 6

Rate of Return (ROR) Analysis

Chapter 6
Rate of Return (ROR) Analysis

General Objective:

Trainee will be able to understand the Rate of Return (ROR) Analysis

Estimated time for this chapter: 4 Training hours

Detailed Objectives:

1. Interpretation of a ROR value.
2. ROR calculation using a PW or AW relation.
3. Using ROR analysis to evaluate a single project.

Introduction

The most commonly quoted measure of economic worth for a project or alternative is its rate of return (ROR). Whether it is an engineering project with cash flow estimates or an investment in a stock or bond, the rate of return is a well-accepted way of determining if the project or investment is economically acceptable. Compared to the PW or AW value, the ROR is a generically different type of measure of worth, as is discussed in this chapter. Correct procedures to calculate a rate of return using a PW or AW relation are explained here, as are some cautions necessary when the ROR technique is applied to a single project's cash flows.

6.1. Interpretation of a ROR value

From the perspective of someone who has borrowed money, the interest rate is applied to the *unpaid balance* so that the total loan amount and interest are paid in full exactly with the last loan payment. From the perspective of a lender of money, there is an *unrecovered balance* at each time period. The interest rate is the return on this unrecovered balance so that the total amount lent and the interest are recovered exactly with the last receipt. *Rate of return* describes both of these perspectives.

Rate of return (ROR) is the rate paid on the unpaid balance of borrowed money, or the rate earned on the unrecovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with interest considered.

The rate of return is expressed as a percent per period, for example, $i = 10\%$ per year. It is stated as a positive percentage; the fact that interest paid on a loan is actually a negative rate of return from the borrower's perspective is not considered. The numerical value of i can range from -100% to infinity, that is, $-100\% \leq i < \infty$. In terms of an investment, a return of $i = -100\%$ means the entire amount is lost.

The definition above does not state that the rate of return is on the initial amount of the investment; rather it is on the **unrecovered balance,** which changes each time period. Example (6-1) illustrates this difference.

Example (6-1)

To get started in a new telecommuting position with AB Hammond Engineers, Jane took out a \$1000 loan at $i = 10\%$ per year for 4 years to buy home office equipment. From the lender's perspective, the investment in this young engineer is expected to produce an equivalent net cash flow of \$315.47 for each of 4 years.

$$A = \$1000(A/P, 10\%, 4) = \$315.47$$

This represents a 10% per year rate of return on the unrecovered balance. Compute the amount of the unrecovered investment for each of the 4 years using

1- The rate of return on the unrecovered balance (the correct basis)
2- The return on the initial \$1000 investment.
3- Explain why all of the initial \$1000 amount is not recovered by the final payment in part (2).

Solution

1- Table (6–1) shows the unrecovered balance at the end of each year in column 6 using the 10% rate on the unrecovered balance at the beginning of the year. After 4 years the total \$1000 is recovered, and the balance in column 6 is exactly zero.

Table (6-1): Unrecovered Balances Using a Rate of Return of 10% on the Unrecovered Balance

(1) Year	(2) Beginning Unrecovered Balance	(3) = 0.10×(2) Interest on Unrecovered Balance	(4) Cash Flow	(5) = (4) – (3) Recovered Amount	(6) = (2) + (5) Ending Unrecovered Balance
0	—	—	$-1000.00	—	$-1000.00
1	$-1000.00	$1000.00	+315.47	$215.47	-784.53
2	-784.53	78.45	+315.47	237.02	-547.51
3	-547.51	54.75	+315.47	260.72	-286.79
4	-286.79	28.68	+315.47	286.79	0
		$261.88		$1000.00	

Table (6-2): Unrecovered Balances Using a 10% Return on the Initial Amount

(1) Year	(2) Beginning Unrecovered Balance	(3) = 0.10×(2) Interest on Initial Amount	(4) Cash Flow	(5) = (4) – (3) Recovered Amount	(6) = (2) + (5) Ending Unrecovered Balance
0	—	—	$-1000.00	—	$-1000.00
1	$-1000.00	$100	+315.47	$215.47	-784.53
2	-784.53	100	+315.47	215.47	-569.06
3	-569.06	100	+315.47	215.47	-353.59
4	-353.59	100	+315.47	215.47	-138.12
		$400		$861.88	

2- Table 7–2 shows the unrecovered balance if the 10% return is always figured on the *initial $1000.* Column 6 in year 4 shows a remaining unrecovered amount of $138.12, because only $861.88 is recovered in the 4 years (column 5).

3- As shown in column 3, a total of $400 in interest must be earned if the 10% return each year is based on the initial amount of $1000. However, only $261.88 in interest must be earned if a 10% return on the unrecovered balance is used. There is more of the annual cash flow available to reduce the remaining loan when the rate is applied to the unrecovered balance as in part (1) and Table (6–1) . Figure (6–1) illustrates the correct interpretation of rate of return in Table (6–1).

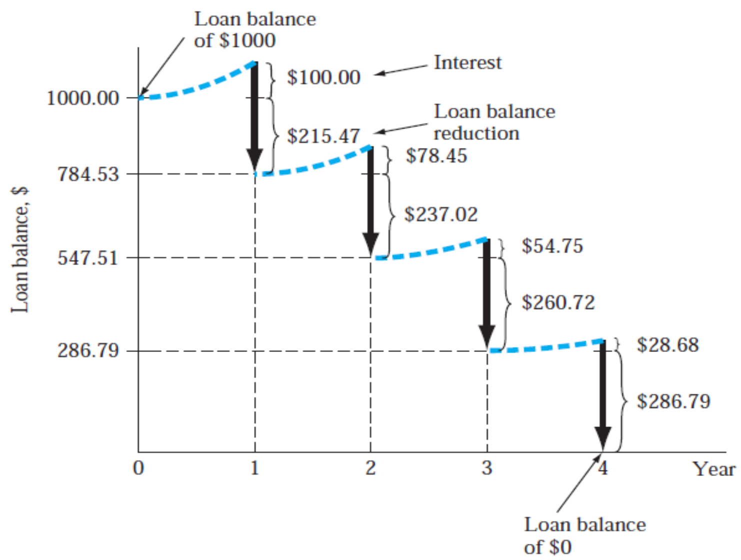


Figure (6-1): Plot of unrecovered balances and 10% per year rate of return on a $1000 amount, Table (6-1).

Each year the $315.47 receipt represents 10% interest on the unrecovered balance in column 2 plus the recovered amount in column 5.

Because rate of return is the interest rate on the unrecovered balance, the computations in *Table (6–1) for part (1) present a correct interpretation of a 10% rate of return.* Clearly, an interest rate applied only to the principal represents a higher rate than is stated. In practice, a so-called add-on interest rate is frequently based on principal only, as in part (2). This is sometimes referred to as the *installment financing* problem.

Installment financing can be discovered in many forms in everyday finances. One popular example is a "no-interest program" offered by retail stores on the sale of major appliances, audio and video equipment, furniture, and other consumer items. Many variations are possible, but in most cases, if the purchase is not paid for in full by the time the promotion is over, usually 6 months to 1 year later, *finance charges are assessed from the original date of purchase.* Further, the program's fine print may stipulate that the purchaser use a credit card issued by the retail company, which often has a higher interest rate than that of a regular credit card, for example, 24% per year compared to 15% per year. In all these types of programs, the one common theme is more interest paid over time by the consumer. Usually, the correct definition of *i* as interest on the unpaid balance does not apply directly; *i* has often been manipulated to the financial disadvantage of the purchaser.

6.2. ROR calculation using a PW or AW relation

The ROR value is determined in a generically different way compared to the PW or AW value for a series of cash flows. For a moment, consider only the present worth relation for a cash flow series. Using the MARR, which is established independent of any particular project's cash flows, a mathematical relation determines the PW value in actual monetary units, say, dollars or euros. For the ROR values calculated in this and later sections, **only the cash flows themselves** are used to determine an interest rate that balances the present worth relation. Therefore, ROR may be considered a relative measure, while PW and AW are absolute measures. Since the resulting interest rate depends only on the cash flows themselves, the correct term is **internal rate of return (IROR);** however, the term *ROR* is used interchangeably. Another definition of rate of return is based on our previous interpretations of PW and AW.

The rate of return is the interest rate that makes the present worth or annual worth of a cash flow series exactly equal to 0.

To determine the rate of return, develop the ROR equation using either a PW or AW relation, set it equal to 0, and solve for the interest rate. Alternatively, the present worth of cash outflows (costs and disbursements) PW_O may be equated to the present worth of cash inflows (revenues and savings) PW_I . That is, solve for *i* using either of the relations

$$0 = PW \qquad Or \qquad PW_O = PW_I \qquad (6-1)$$

The annual worth approach utilizes the AW values in the same fashion to solve for *i* .

$$0 = AW \qquad Or \qquad AW_O = AW_I \qquad (6-2)$$

The *i* value that makes these equations numerically correct is called *i**. It is the root of the ROR relation. To determine if the investment project's cash flow series is viable, compare *i** with the established MARR.

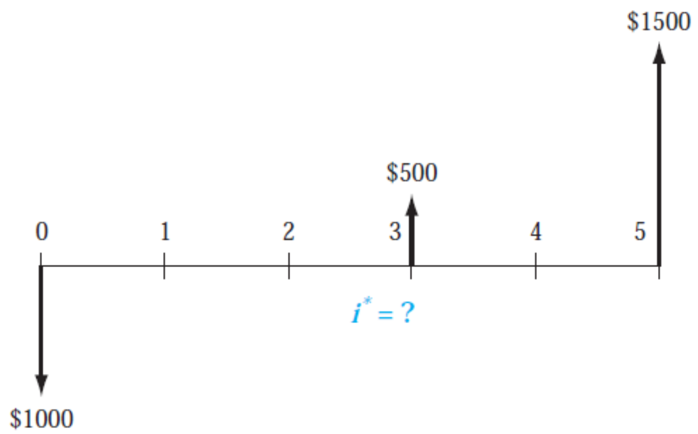


Figure (6-2): Cash flow for which a value of *i* is to be determined.

The guideline is as follows:

If i* ≥ MARR, accept the project as economically viable.
If i* ≥ MARR, the project is not economically viable.

The purpose of engineering economy calculations is *equivalence* in PW or AW terms for a stated i ≥ 0%. In rate of return calculations, the objective is to *find the interest rate i** at which the cash flows are equivalent. The calculations are the reverse of those made in previous chapters, where the interest rate was known. For example, if you deposit $1000 now and are promised payments of $500 three years from now and $1500 five years from now, the rate of return relation using PW factors and Equation (6-1) is

$$1000 = 500(P/F, i^*, 3) \ = 1500(P/F, i^*, 5)$$

The value of *i** that makes the equality correct is to be determined (see Figure (6–2)). If the $1000 is moved to the right side of Equation (6-3), we have the form 0 = PW.

$$0 = -1000 + 500(P/F, i^*, 3) \ = 1500(P/F, i^*, 5)$$

The equation is solved for *i* * _ 16.9% by hand using trial and error or using a spreadsheet function. The rate of return will always be greater than zero if the total amount of cash inflow is greater than the total amount of outflow, when the time value of money is considered. Using *i** = 16.9%, a graph similar to Figure (6–1) can be constructed. It will show that the unrecovered balances each year, starting with $ -1000 in year 1, are exactly recovered by the $500 and $1500 receipts in years 3 and 5.

It should be evident that rate of return relations are merely a rearrangement of a present worth equation. That is, if the above interest rate is known to be 16.9%, and it is used to find the present worth of $500 three years from now and $1500 five years from now, the PW relation is

$$PW = 500(P/F, 16.9\%, 3) \ + 1500(P/F, 9\%, 5) = \$1000$$

This illustrates that rate of return and present worth equations are set up in exactly the same fashion. The only differences are what is given and what is sought.

There are several ways to determine *i** once the PW relation is established: solution via trial and error by hand, using a programmable calculator, and solution by spreadsheet function. The spreadsheet is faster; the first helps in understanding how ROR computations work. We summarize two methods here and in Example (6-2).

i* Using Trial and Error The general procedure of using a PW-based equation is as follows:

1- Draw a cash flow diagram.
2- Set up the rate of return equation in the form of Equation (6-1).
3- Select values of *i* by trial and error until the equation is balanced.

When the trial-and-error method is applied to determine *i**, it is advantageous in step 3 to get fairly close to the correct answer on the first trial. If the cash flows are combined in such a manner that the income and disbursements can be represented by a *single factor* such as *P*/*F* or *P*/*A* , it is possible to look up the interest rate (in the tables) corresponding to the value of that factor for *n* years. The problem, then, is to combine the cash flows into the format of only one of the factors. This may be done through the following procedure:

1- Convert all *disbursements* into either single amounts (*P* or *F*) or uniform amounts (*A*) by neglecting the time value of money. For example, if it is desired to convert an *A* to an *F* value, simply multiply the *A* by the number of years *n*. The scheme selected for movement of cash flows should be the one that minimizes the error caused by neglecting the time value of money. That is, if most of the cash flow is an *A* and a small amount is an *F* , convert the *F* to an *A* rather than the other way around.
2- Convert all *receipts* to either single or uniform values.
3- Having combined the disbursements and receipts so that a *P*/*F*, *P*/*A*, or *A*/*F* format applies, use the interest tables to find the approximate interest rate at which the *P*/*F*, *P*/*A*, or *A*/*F* value is satisfied. The rate obtained is a good estimate for the first trial.

It is important to recognize that this first-trial rate is only an *estimate* of the actual rate of return, because the time value of money is neglected. The procedure is illustrated in Example (6-2).

i* by Spreadsheet The fastest way to determine an *i** value when there is a series of equal cash flows (*A* series) is to apply the RATE function. This is a powerful one-cell function, where it is acceptable to have a separate *P* value in year 0 and a separate *F* value in year *n* . The format is

$$= RATE\ (n, A, P, F) \qquad (6-4)$$

When cash flows vary from year to year (period to period), the best way to find *i** is to enter the net cash flows into contiguous cells (including any \$0 amounts) and apply the IRR function in any cell. The format is

$$= IRR(first\_cell{:}\,last\_cell, guess) \qquad (6-5)$$

Where “guess” is the *i* value at which the function starts searching for *i**. The PW-based procedure for sensitivity analysis and a graphical estimation of the *i** value is as follows:

1- Draw the cash flow diagram.
2- Set up the ROR relation in the form of Equation (6-1), PW = 0.
3- Enter the cash flows onto the spreadsheet in contiguous cells.
4- Develop the IRR function to display i*.
5- Use the NPV function to develop a PW graph (PW versus *i* values). This graphically shows the *i** value at which PW = 0.

Example (6-2)

Applications of green, lean manufacturing techniques coupled with value stream mapping can make large financial differences over future years while placing greater emphasis on environmental factors. Engineers with Monarch Paints have recommended to management an investment of \$200,000 now in novel methods that will reduce the amount of wastewater, packaging materials, and other solid waste in their consumer paint manufacturing facility. Estimated savings are \$15,000 per year for each of the next 10 years and an additional savings of \$300,000 at the end of 10 years in facility and equipment upgrade costs. Determine the rate of return using hand and spreadsheet solutions.

Solution

Solution by Hand

Use the trial-and-error procedure based on a PW equation.

1- Figure (6–3) shows the cash flow diagram.

2- Use Equation (6-1) format for the ROR equation.

$$0 = -200{,}000 + 15{,}000(P/A, i^*, 10) + 300{,}000(P/F, i^*, 10) \qquad (6-6)$$

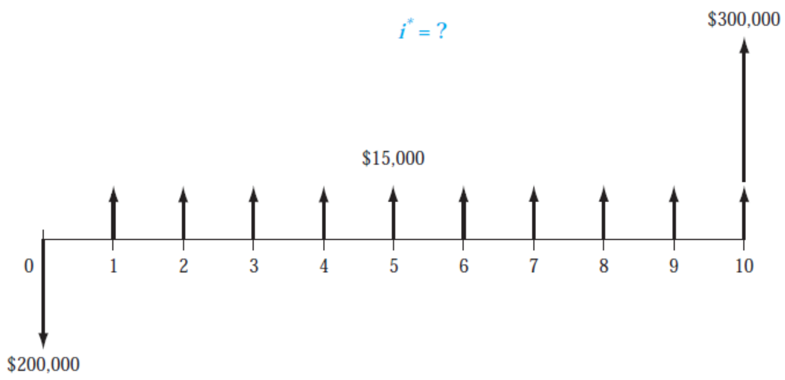


Figure (6-3): Cash flow diagram, Example (6-2).

3- Use the estimation procedure to determine i for the first trial. All income will be regarded as a single *F* in year 10 so that the *P*/*F* factor can be used. The *P*/*F* factor is selected because most of the cash flow (\$300,000) already fits this factor and errors created by neglecting the time value of the remaining money will be minimized. Only for the first estimate of *i*, define *P* = \$200,000, *n* = 10, and *F* = 10(15,000) + 300,000 = \$450,000.
Now we can state that

$$200{,}000 = 450{,}000(P/F, i, 10); \quad (P/F, i, 10) = 0.44$$

The roughly estimated *i* is between 8% and 9%. Use 9% as the first trial because this approximate rate for the *P*/*F* factor will be lower than the true value when the time value of money is considered.

$$0 = -200{,}000 + 15{,}000(P/A, 9\%, 10) + 300{,}000(P/F, 9\%, 10)$$

$$0 < \$22{,}986$$

The result is positive, indicating that the return is more than 9%. Try *i* = 11%.

$$0 = -200{,}000 + 15{,}000(P/A, 11\%, 10) + 300{,}000(P/F, 11\%, 10)$$

$$0 < \$ - 6002$$

Since the interest rate of 11% is too high, linearly interpolate between 9% and 11%.

$$i^* = 9.00 + \frac{22.986 - 0}{22 - 986 - (-6002)}(2.0)$$

$$= 9.00 + 1.58 = 10.58\%$$

Solution by Spreadsheet

The fastest way to find *i** is to use the RATE function (Equation (6-4)). The entry = RATE(10,15000,-200000,300000) displays *i** = 10.55% per year. It is equally correct to use the IRR function. Figure (6-4) , column B, shows the cash flows and = IRR(B2:B12) function to obtain *i**.

For a complete spreadsheet analysis, use the procedure outlined above.

1- Figure (6-3) shows cash flows.
2- Equation (6-6) is the ROR relation.
3- Figure (6–4) shows the net cash flows in column B.
4- The IRR function in cell B14 displays *i** = 10.55%.
5- To graphically observe *i** = 10.55%, column D displays the PW graph for different *i* values. The NPV function is used repeatedly to calculate PW for the *xy* scatter chart.

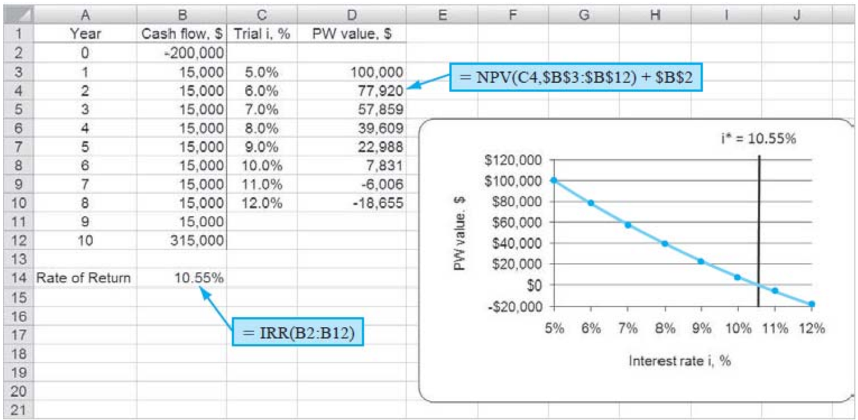


Figure (6-4): Spreadsheet to determine i^* and develop a PW graph, Example (6-2).

Just as *i** can be found using a PW equation, it may equivalently be determined using an AW relation. This method is preferred when uniform annual cash flows are involved. Solution by hand is the same as the procedure for a PW-based relation, except Equation (6-2) is used. In the case of Example (6-2) , i^* = 10.55% is determined using the AW-based relation.

$$0 = -200{,}000(A/P, i^*, 10) + 15{,}000 + 300{,}000(A/F, i^*, 10)$$

The procedure for solution by spreadsheet is exactly the same as outlined above using the IRR function. Internally, IRR calculates the NPV function at different *i* values until NPV = 0. (There is no equivalent way to utilize the PMT function, since it requires a fixed value of *i* to calculate an *A* value.)

6.3. Using ROR analysis to evaluate a single project

The rate of return method is commonly used in engineering and business settings to evaluate one project, as discussed in this chapter, and to select one alternative from two or more, as explained in the next chapter. As mentioned earlier, an ROR analysis is performed using a different basis than PW and AW analyses. The cash flows themselves determine the (internal) rate of return. As a result, there are some assumptions and special considerations with ROR analysis that must be made when calculating *i** and in interpreting its real-world meaning. A summary is provided below.

- ***Multiple i* values.*** Depending upon the sequence of net cash inflows and outflows, there may be more than one real-number root to the ROR equation, resulting in *more than one i* value.*
- ***Reinvestment at i*.*** Both the PW and AW methods assume that any net positive investment (i.e., net positive cash flows once the time value of money is considered) is reinvested at the MARR. However, the ROR method assumes reinvestment at the i^* rate. When i^* is not close to the MARR (e.g., if i^* is substantially larger than MARR), this is an unrealistic assumption. In such cases, the i^* value is not a good basis for decision making.
- ***Different procedure for multiple alternative evaluations.*** To correctly use the ROR method to choose from two or more mutually exclusive alternatives requires an *incremental analysis* procedure that is significantly more involved than PW and AW analysis.

If possible, from an engineering economic study perspective, the **AW or PW method at a stated MARR should be used in lieu of the ROR method** . However, there is a strong appeal for the ROR method because rate of return values are very commonly quoted. And it is easy to compare a proposed project's return with that of in-place projects.

When it is important to know the exact value of i*, a good approach is to determine PW or AW at the MARR, then determine the specific i* for the selected alternative.

As an illustration, if a project is evaluated at MARR =15% and has PW< 0, there is no need to calculate i^*, because $i^*<15\%$. However, if PW is positive, but close to 0, calculate the exact i^* and report it along with the conclusion that the project is financially justified.

Problems

1- A shrewd investor loaned $1,000,000 to a start-up company at 10% per year interest for 3 years, but the terms of the agreement were such that interest would be charged on the principal rather than on the unpaid balance. How much extra interest did the company pay?

2- What is the *nominal* rate of return per year on an investment that increases in value by 8% every 3 months?

3- Assume you borrow $50,000 at 10% per year interest and you agree to repay the loan in five equal annual payments. What is the amount of the unrecovered balance immediately after you make the third payment?

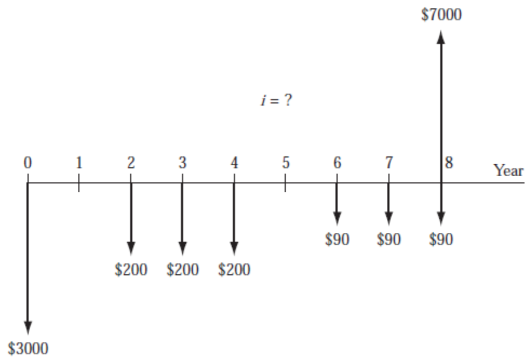
4- International Potash got a $50 million loan amortized over a 10-year period at 10% per year interest. The loan agreement stipulates that the loan will be repaid in 10 equal annual payments with interest charged on the principal amount of the loan (not on the unrecovered balance).

c) What is the amount of each payment?
d) What is the total amount of interest paid?
e) How does the total interest paid compare with the principal of the loan?

5- In 2010, the city of Houston, Texas, collected $24,112,054 in fines from motorists because of traffic violations caught by red-light cameras. The cost of operating the system was $8,432,372. The net profit, that is, profit after operating costs, is split equally (that is, 50% each) between the city and the operator of the camera system. What will be the rate of return over a 3-year period to the contractor that paid for, installed, and operates the system, if its initial cost was $9,000,000 and the profit for each of the 3 years is the same as it was in 2010?

6- P&G sold its prescription drug business to Warner-Chilcott, Ltd. for $3.1 billion. If income from product sales is $2 billion per year and net profit is 20% of sales, what rate of return will the company make over a 10-year planning horizon?

7- Determine the rate of return for the cash flows shown in the diagram. (If requested by your instructor, show both hand and spreadsheet solutions.)



8- The Ester Municipal Water Utility issued 20-year bonds in the amount of $53 million for several high-priority flood control improvement projects. The bonds carried a 5.38% dividend rate with the dividend payable annually. The U.S. economy was in a recession at that time, so as part of the federal stimulus program, the Utility gets a 35% reimbursement on the dividend it pays.

a) What is the effective dividend rate that the Utility is paying on the bonds?
b) What is the total dollar amount the Utility will *save* in dividends over the life of the bonds?

c) What is the future worth in year 20 of the dividend savings, if the interest rate is 6% per year?

9- For the cash flows shown, determine the rate of return.

Year	0	1	2	3	4	5
Expense, $	-17,000	-2,500	-2,500	-2,500	-2,500	-2,500
Revenue, $	0	5,000	6,000	7,000	8,000	12,000

Chapter 7

Benefit/Cost Analysis and Public-Sector Economics

Chapter 7
Benefit/Cost Analysis and Public-Sector Economics

General Objective:

Trainee will be able to understand the Benefit/Cost Analysis and Public-Sector Economics

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

1. The Fundamental Differences Between Public and Private Sector projects.
2. Benefit/Cost Analysis of a Single Project.

Introduction

The evaluation methods of previous chapters are usually applied to alternatives in the private sector, that is, for-profit and not-for-profit corporations and businesses. This chapter introduces public sector alternatives and their economic consideration.

7.1. The Fundamental Differences Between Public and Private Sector projects.

7.1.1. Public Sector Project Description

Public sector projects have a primary purpose to provide services to the citizenry for the public good at no profit. Areas such as health, transportation, safety, economic development, and utilities comprise a majority of alternatives that require engineering economic analysis. Some public sector examples are:

Hospitals and clinics
Parks and recreation
Utilities: water, electricity, gas, sewer, sanitation
Schools: primary, secondary, community colleges, universities
Economic development
Convention centers
Sports arenas
Transportation: highways, bridges, waterways
Police and fire protection
Courts and prisons
Food stamp and rent relief programs
Job training
Public housing
Emergency relief
Codes and standards

There are significant differences in the characteristics of private and public sector alternatives.

Characteristic	***Public Sector***	***Private Sector***
Size of investment	Larger	Some large; more medium to small

Often alternatives developed to serve public needs require large initial investments, possibly distributed over several years. Modern highways, public transportation systems, airports, and flood control systems are examples.

Life estimates	Longer (30–50+ years)	Shorter (2–25 years)

The long lives of public projects often prompt the use of the capitalized cost method, where infinity is used for n and annual costs are calculated as $A = P(i)$.

Annual cash flow estimates	No profit; costs, benefits, and disbenefits are estimated	Revenues and savings contribute to profits; costs are estimated

Public sector projects (also called publicly owned) do not have profits; they do have costs paid by the appropriate government unit; and they benefit the citizenry. Public sector projects often have undesirable consequences (disbenefits). It is these consequences that can cause public controversy about the projects, because benefits to one group of taxpayers might be disbenefits to other taxpayers, as discussed more fully below. Due to the subjective nature of public project data, it is relatively easy to manipulate estimates; however, to perform an economic analysis of public alternatives, the costs (initial and annual), the benefits, and the disbenefits, if considered, must be estimated as accurately as possible in monetary units.

- **Costs:** estimated expenditures to the government entity for construction, operation, and maintenance of the project, less any expected salvage value.

- **Benefits:** advantages to be experienced by the owners, the public. Benefits can include revenue and savings.
- **Disbenefits:** expected undesirable consequences that are indirect economic disadvantages to the owners if the alternative is implemented.

In many cases, it is difficult to estimate and agree upon the economic impact of benefits and disbenefits for a public-sector alternative. For example, assume a short bypass around a congested traffic area is recommended. How much will it benefit a driver in dollars per driving minute to be able to bypass five traffic lights, as compared to stopping at an average of two lights for 45 seconds each? The bases and standards for benefits estimation are always difficult to establish and verify. Relative to revenue cash flow estimates in the private sector, benefit estimates are much harder to make, and they vary more widely around uncertain averages. And the disbenefits that accrue from an alternative are harder to estimate.

The examples in this chapter include straightforward identification of benefits, disbenefits, and costs. However, in actual situations, judgments are subject to interpretation, particularly in determining which elements of cash flow should be included in the economic evaluation. For example, improvements to the condition of pavement on city streets might result in fewer accidents, an obvious benefit to the taxpaying public. But fewer damaged cars and personal injuries mean less work and money for auto repair shops, towing companies, car dealerships, doctors and hospitals—also part of the taxpaying public. It may be necessary to take a limited viewpoint, because in the broadest viewpoint benefits are usually offset by approximately equal disbenefits.

Funding	Taxes, fees, bonds, private funds	Stocks, bonds, loans, individual owners

The capital used to finance public sector projects is commonly acquired from taxes, bonds, fees, and gifts from private donors. Taxes are collected from those who are the owners—the citizens (e.g., gasoline taxes for highways are paid by all gasoline users). This is also the case for fees, such as toll road fees for drivers. Bonds are often issued: municipal bonds and special-purpose bonds, such as utility district bonds.

Interest rate	Lower	Higher, based on market cost of capital

The interest rate for public sector projects, also called the discount rate, is virtually always lower than for private sector alternatives. Government agencies are exempt from taxes levied by higher-level government units. For example, municipal projects do not have to pay state taxes. Also, many loans are government subsidized and carry low interest rates.

Selection criteria	Multiple criteria	Primarily based on MARR

Multiple categories of users, economic as well as noneconomic interests, and special interest political and citizen groups make the selection of one alternative over another much more difficult in public sector economics. Seldom is it possible to select an alternative on the sole basis of a criterion such as PW or ROR.

Environment of the evaluation	Politically inclined	Primarily economic

There are often public meetings and debates associated with public sector projects. Elected officials commonly assist with the selection, especially when pressure is brought to bear by voters, developers, environmentalists, and others. The selection process is not as "clean" as in private sector evaluation.

The viewpoint of the public sector analysis must be determined before cost, benefit, and disbenefit estimates are made. There are always several viewpoints that may alter how a cash flow estimate is classified. Some example viewpoints are the citizen; the tax base; number of students in the school district; creation and retention of jobs; economic development potential; or a particular industry interest. Once established, the viewpoint assists in categorizing the costs, benefits, and disbenefits of each alternative.

Example (7-1)

The citizen-based Capital Improvement Projects (CIP) Committee for the city of Dundee has recommended a $25 million bond issue for the purchase of greenbelt/floodplain land to preserve low-lying green areas and wildlife habitat. Developers oppose the proposal due to the reduction of available land for commercial development. The city engineer and economic development director have made preliminary estimates for some obvious areas over a projected 15-year planning horizon. The inaccuracy of these estimates is made very clear in a report to the Dundee City Council. The estimates are not yet classified as costs, benefits, or disbenefits.

Economic Dimension	Estimate
Annual cost of $5 million in bonds over 15 years at a 6% bond interest rate	$300,000 (years 1–14) $5,300,000 (year 15)
Annual maintenance, upkeep, and program management	$75,000 +10% per year increase
Annual parks development budget	$500,000 (years 5–10)
Annual loss in commercial development	$2,000,000 (years 8–10)
State sales tax rebates not realized	$275,000 + 5% per year (years 8 on)
Annual municipal income from park use and regional sports events	$100,000 + 12% per year (years 6 on)
Savings in flood control projects	$300,000 (years 3–10) $1,400,000 (years 11–15)
Property damage (personal and city) not incurred due to flooding	$500,000 (years 10 and 15)

Identify different viewpoints for an economic analysis of the proposal and classify the estimates accordingly.

Solution

There are many perspectives to take; three are addressed here. The viewpoints and goals are identified and each estimate is classified as a cost, benefit, or disbenefit. (How the classification is made will vary depending upon who does the analysis. This solution offers only one logical answer.)

Viewpoint 1: Citizen of the city. Goal: Maximize the quality and wellness of citizens with family and neighborhood as prime concerns.

Costs: 1, 2, 3 Benefits: 6, 7, 8 Disbenefits: 4, 5

Viewpoint 2: City budget. Goal: Ensure the budget is balanced and of sufficient size to fund rapidly growing city services.

Costs: 1, 2, 3,5 Benefits: 6, 7, 8 Disbenefits: 4

Viewpoint 3: Economic development. Goal: Promote new commercial and industrial economic development for creation and retention of jobs.

Costs: 1, 2, 3, 4, 5 Benefits: 6, 7, 8 Disbenefits: none

7.1.2. Ethics Considerations

Engineers are involved in a wide range of public sector activities, which may be generally classified into two categories—policy and planning.

- **Policy making**: These activities include *strategy development* based upon feasibility studies, surveys, historical precedent, legal requirement, current data, and hypothesis testing. Examples are highway and air transportation management and healthcare systems policies. In the case of highway transportation, contract and government-employed engineers make most of the recommendations on elements such as highway expansion, routing, capacity, zoning, speed limits, and traffic signaling systems.
- **Planning**: This includes *project development and oversight* that implement approved policies and strategies. These projects affect people, the environment, and finances. The planning level of examples mentioned above could be healthcare delivery methods, highway traffic control, and air traffic control projects. In the case of highway traffic control, engineers implement policy through plans for commercial and residential corridors, speed control, and monitoring projects (e.g., camera surveillance), parking restrictions, traffic light placement, toll roads, etc.

In all of these types of activities, the public expects its engineers, like elected officers and politicians, to have good morals and ethics. Among other things, they are expected to:

- Demonstrate high standards
- Make realistic assumptions and conclusions
- Gather and use data and information fairly
- Be impartial in decision making
- Consider a wide range of circumstances before deciding on a particular strategy or plan

In other words, it is assumed that public servants of all types (elected, employed, and contracted) will have and demonstrate *integrity and fairness* in all of their dealings. Most citizens become very disappointed and discouraged in the public leadership, including engineers, when these qualities are compromised. It is vitally important that engineers maintain adherence to the Code of Ethics for Engineers and stay clear of potentially unethical practices. However, it is quite easy to become involved in situations that involve a public-sector project that offers ethical challenges. Here is one simple example.

- ***The situation:*** Joe, a city of Vickor engineer employed by the energy department, is currently full time on a project that involves the conversion of all residential electrical meters from read-on-site to read-remotely. Some 95,000 meters will be replaced at an expenditure of several million dollars by a contractor to be chosen. Over the last few weeks, Joe has become a close friend of Lisa, who, he has learned, works as a proposal writer for Lange Contractors, one of the two or three companies expected to bid on the meter replacement contract.
- ***The temptation:*** In his efforts to impress Lisa, Joe has thought of mentioning to Lisa that his boss will cast a major vote when the meter-replacement contract is awarded, and that he, Joe's manager, is very positively impressed with the meters manufactured by Hammond Industries. In fact, Joe's manager has stated privately to Joe that a proposing contractor will have an edge for his vote if Hammond meters are specified in the equipment portion of the proposal.
- ***The ethics dilemma:*** As an experienced engineer, Joe realizes he must not provide Lisa with this sort of advantage when writing the proposal. Though he may be tempted to provide "hints" if the conversation turns to work topics, to remain professionally ethical, Joe must refrain from any sort of information that will favor the Lange proposal. This is ethically correct even if Joe learns that Lange is already planning to

propose meters manufactured by Hammond Industries. On the other side, Lisa should not ask Joe about the proposal or share any of its contents during their conversations.

7.2. Benefit/Cost Analysis of a Single Project.

The benefit/cost ratio, a fundamental analysis method for public sector projects, was developed to introduce more objectivity into public sector economics. It was developed in response to the U.S. Flood Control Act of 1936. There are several variations of the B/C ratio; however, the basic approach is the same. All cost and benefit estimates must be converted to a common equivalent monetary unit (PW, AW, or FW) at the discount rate (interest rate). The B/C ratio is then calculated using one of these relations:

$$B/C=\frac{PW\ of\ benefits}{PW\ of\ costs}=\frac{AW\ of\ benefits}{AW\ of\ costs}=\frac{FW\ of\ benefits}{FW\ of\ costs} \qquad (7-1)$$

The sign convention for B/C analysis is positive signs, so costs are preceded by a + sign. Salvage values, when they are estimated, are subtracted from costs. Disbenefits are considered in different ways depending upon the model used. Most commonly, disbenefits are subtracted from benefits and placed in the numerator. The different formats are discussed below. The decision guideline for a single project is simple:

If B/C ≥ 1.0, accept the project as economically acceptable for the estimates and discount rate applied.

If B/C < 1.0, the project is not economically acceptable.

The conventional B/C ratio is the most widely used. It subtracts disbenefits from benefits.

$$B/C=\frac{benefits-dibenefits}{costs}=\frac{B-D}{C} \qquad (7-2)$$

The B/C value would change considerably were disbenefits added to costs. For example, if the numbers 10, 8, and 5 are used to represent the PW of benefits, disbenefits, and costs, respectively, Equation (7-2) results in B/C = (10 - 8)/5 = 0.40. The incorrect placement of disbenefits in the denominator results in B/C = 10/(8 + 5) = 0.77, which is approximately twice the correct B/C value. Clearly, then, the method by which disbenefits are handled affects the magnitude of the B/C ratio. However, it does not matter whether disbenefits are (correctly) subtracted from the numerator or (incorrectly) added to costs in the denominator, a B/C ratio of less than 1.0 by the first method will always yield a B/C ratio less than 1.0 by the second method, and vice versa. The modified B/C ratio places benefits (including revenues and savings), disbenefits, and maintenance and operation (M&O) costs in the numerator. The denominator includes only the equivalent PW, AW, or FW of the initial investment.

$$Modified\ B/C=\frac{benefits-dibenefits-M\&O\ costs}{initial\ investment} \qquad (7-3)$$

Salvage value is included in the denominator with a negative sign. The modified B/C ratio will obviously yield a different value than the conventional B/C method. However, as discussed above with disbenefits, the modified procedure can change the magnitude of the ratio but not the decision to accept or reject the project. The decision guideline remains the same; if the modified B/C ratio exceeds or equals 1.0, the project is justified.

The benefit and cost difference measure of worth, which does not involve a ratio, is based on the difference between the PW, AW, or FW of benefits (including income and savings) and costs, that is, B - C. If (B - C) ≥ 0, the project is acceptable. This method has the advantage of eliminating the discrepancies noted above when disbenefits are regarded as costs, because B represents net benefits. Thus, for the numbers 10, 8, and 5 the same result is obtained regardless of how disbenefits are treated.

Subtracting disbenefits from benefits: $B / C = (10 - 8) - 5 = -3$

Adding disbenefits to costs: $B / C = 10 - (8 + 5) = -3$

Example (7-2)

The Ford Foundation expects to award $15 million in grants to public high schools to develop new ways to teach the fundamentals of engineering that prepare students for university-level material. The grants will extend over a 10-year period and will create an estimated savings of $1.5 million per year in faculty salaries and student related expenses. The Foundation uses a discount rate of 6% per year. This grants program will share Foundation funding with ongoing activities, so an estimated $200,000 per year will be removed from other program funding. To make this program successful, a $500,000 per year operating cost will be incurred from the regular M&O budget. Use the B/C method to determine if the grants program is economically justified.

Solution

Use annual worth as the common monetary equivalent. For illustration only, all three B/C models are applied.

AW of investment cost. $\$15{,}000{,}000(A/P, 6\%, 10) = \$2{,}038{,}050\ per\ year$

AW of M&O cost. $\$500{,}000\ per\ year$

AW of benefit. $\$1{,}500{,}000\ per\ year$

AW of disbenefit. $\$200{,}000\ per\ year$

Use Equation (7-2) for conventional B/C analysis, where M&O is placed in the denominator as an annual cost. The project is not justified, since B/C < 1.0.

$$B/C = \frac{1{,}500{,}000 - 200{,}000}{2{,}038{,}050 + 500{,}000} = \frac{1{,}300{,}000}{2{,}538{,}050} = 0.51$$

By Equation (7-3) the modified B/C ratio treats the M&O cost as a reduction to benefits.

$$Modified\ B/C = \frac{1{,}500{,}000 - 200{,}000 - 500{,}000}{2{,}038{,}050} = 0.39$$

For the (B - C) model, B is the net benefit, and the annual M&O cost is included with costs.

$$B/C = (1{,}500{,}000 - 200{,}000) - (2{,}038{,}050 - 500{,}000) = \$ - 1.24\ million$$

Problems

1- In conducting a B/C analysis, why is it best to take a limited viewpoint in determining benefits and disbenefits?

2- Identify the following as primarily public or private sector projects.
  a. Bridge across Ohio River
  b. Coal mine
  c. Baja 1000 race team
  d. Consulting engineering firm
  e. County courthouse
  f. Flood control project
  g. Endangered species designation
  h. Freeway lighting
  i. Antarctic cruise
  j. Crop dusting

3- Identify the following as primarily private or public sector characteristics:
  a. Large investment
  b. No profits
  c. Funding from fees
  d. MARR-based selection criteria
  e. Low interest rate
  f. Short project life estimate
  g. Disbenefits

4- Calculate the conventional B/C ratio for a county government project that is projected to have the following cash flows: costs of $2,000,000 per year; benefits of $2,740,000 per year; disbenefits of $380,000 per year.

5- The following estimates (in $1000 units) have been developed for a new security system at Chicago's O'Hare Airport.

First cost, $	13.000
AW of benefits, $/year	3.800
FW (in year 20) of disbenefits, $	6.750
M&O costs, $/year	400
Life of project, years	20

  a. Calculate the conventional B/C ratio at a discount rate of 10% per year.
  b. Determine the minimum first cost necessary to make the project economically unjustified.

Chapter 8

Breakeven and Payback Analysis

Chapter 8
Breakeven and Payback Analysis

General Objective:

Trainee will be able to understand the Breakeven and Payback Analysis

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

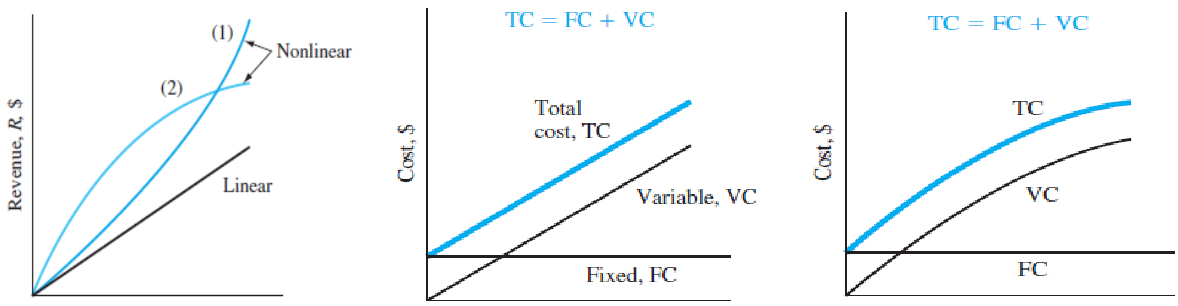
1. Breakeven Analysis for a Single Project.
2. Breakeven Analysis Between Two Alternatives.

Introduction

This chapter covers several related topics that assist in evaluating the effects of varying estimated values for one or more parameters present in an economic study. Since all estimates are for the future, it is important to understand which parameter(s) may make a significant impact on the economic justification of a project. Breakeven analysis is performed for one project or two alternatives. For a single project, it determines a parameter value that makes revenue equal cost. Two alternatives break even when they are equally acceptable based upon a calculated. value of one parameter common to both alternatives. Make-or-buy decisions, also called inhouse-outsource decisions, for most subcontractor services, manufactured components, or international contracts, are routinely based upon the outcome of a breakeven analysis.

8.1. Breakeven Analysis for a Single Project

Public sector projects have a primary purpose to provide services to the citizenry for the Breakeven analysis determines the value of a parameter or decision variable that makes two relations equal. For example, breakeven analysis can determine the required years of use to recover the initial investment and annual operating costs. There are many forms of breakeven analysis; some equate PW or AW equivalence relations, some involve equating revenue and cost relations, others may equate demand and supply relations. However, they all have a common approach, that is, to equate two relations, or to set their difference equal to zero, and solve for the breakeven value of one variable that makes the equation true. The need to determine the breakeven value of a decision variable without including the time value of money is common. For example, the variable may be a design capacity that will minimize cost, or the sales volume necessary to cover costs, or the cost of fuel to maximize revenue from electricity generation. Figure (8-1a) presents different shapes of a revenue relation identified as R. A linear revenue relation is commonly assumed, but a nonlinear relation is often more realistic with increasing per unit revenue for larger volumes (curve 1), or decreasing per unit revenue (curve 2).



(a) Revenue relations linear, increasing (1) and decreasing (2) per unit *(b) Linear cost relations* *(c) Nonlinear cost relations*

Figure (8-1):Linear and nonlinear revenue and cost relations.

Costs, which may be linear or nonlinear, usually include two components fixed and variable as indicated in Figures (8-1b and c).

Fixed cost (FC). Includes costs such as buildings, insurance, fixed overhead, some minimum level of labor, equipment capital recovery, and information systems.

Variable cost (VC). Includes costs such as direct labor, subcontractors, materials, indirect costs, marketing, advertisement, legal, and warranty.

The fixed cost component is essentially constant for all values of the variable, so it does not vary significantly over a wide range of operating parameters. Even if no output is produced, fixed costs are incurred at some threshold level. (Of course, this situation cannot last long before the operation must shut down.) Fixed costs are reduced through improved equipment, information systems and workforce utilization, less costly fringe benefit packages, subcontracting specific functions, and so on. A simple VC relation is vQ, where v is the variable cost per unit and Q is the quantity. Variable costs change with output level, workforce size, and many other parameters. It is usually possible to decrease variable costs through improvements in design, efficiency, automation, materials, quality, safety, and sales volume. When FC and VC are added, they form the total cost relation TC. Figure (8-1b) illustrates linear fixed and variable costs. Figure (8-1c) shows TC for a nonlinear VC in which unit variable costs decrease as Q rises. At some value of Q, the revenue and total cost relations will intersect to identify the breakeven point Q_{BE} (Figure (8-2a)). If $Q > QBE$, there is a profit; but if $Q < Q_{BE}$, there is a loss. For linear R and TC, the greater the quantity, the larger the profit. Profit is calculated as

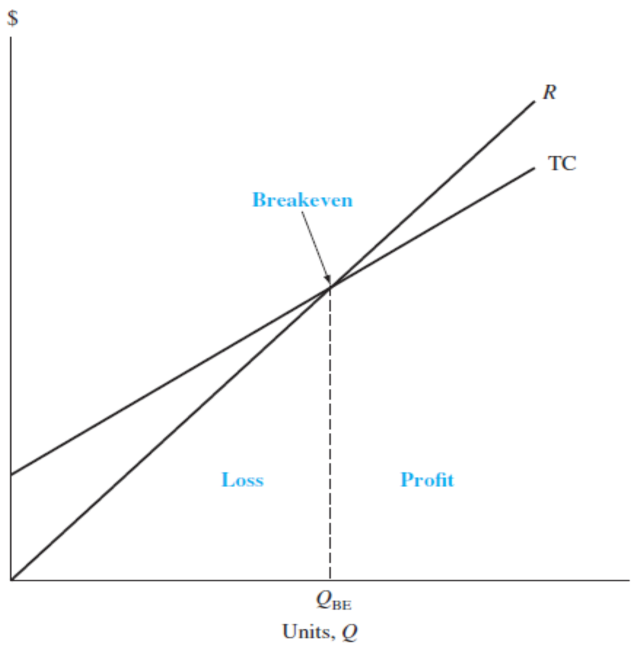
$$Profit = revenue - total\ cost = R - TC \qquad (8-1)$$

A closed-form solution for Q_{BE} may be derived when revenue and total cost are linear functions of Q by equating the relations, indicating a profit of zero.

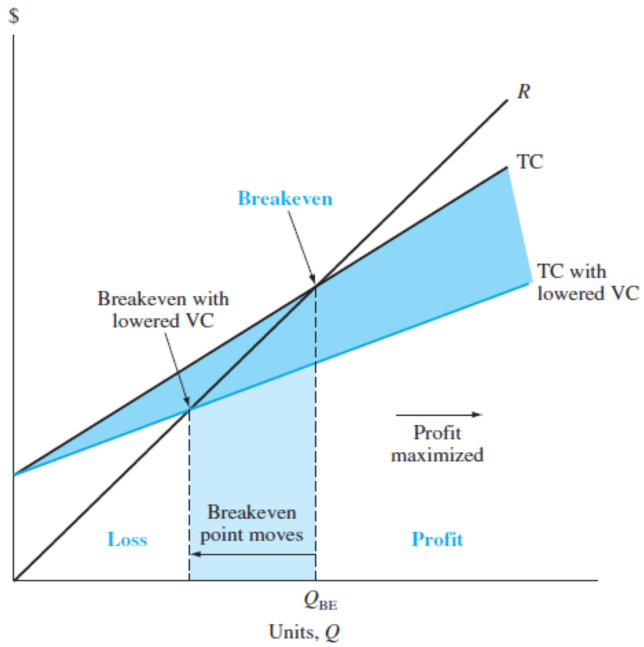
$R = TC$
$rQ = FC + VC = FC + nQ$
where r = revenue per unit
v = variable cost per unit
Solve for Q to obtain the breakeven quantity.

$$Q_{BE} = \frac{FC}{r - v} \qquad (8-2)$$

The breakeven graph is an important management tool because it is easy to understand. For example, if the variable cost per unit is reduced, the linear TC line has a smaller slope (Figure (8-2*b*)), and the breakeven point decreases. This is an advantage because the smaller the value of Q_{BE}, the greater the profit for a given amount of revenue.



(a)



(b)

Figure (8-2): (a) Breakeven point and (b) effect on breakeven point when the variable cost per unit is reduced

If nonlinear R or TC models are used, there may be more than one breakeven point. Figure (8-3) presents this situation for two breakeven points. The maximum profit occurs at Q_P where the distance between the R and TC curves is greatest.

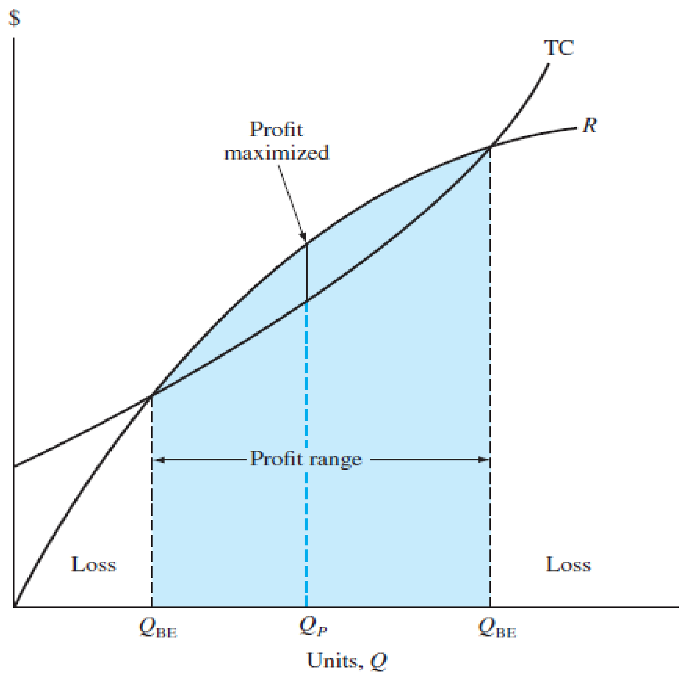


Figure (8-3): Breakeven points and maximum profit point for a nonlinear analysis.

Example (8-1)

Nicholea Water LLC dispenses its product Nature's Pure Water via vending machines with most current locations at food markets and pharmacy or chemist stores. The average monthly fixed cost per site is $900, while each gallon costs 18¢ to purify and sells for 30¢.

a. Determine the monthly sales volume needed to break even.
b. Nicholea's president is negotiating for a sole-source contract with a municipal government where several sites will dispense larger amounts. The fixed cost and purification costs will be the same, but the sales price per gallon will be 30¢ for the first 5000 gallons per month and 20¢ for all above this threshold level. Determine the monthly breakeven volume at each site.

Solution

a. Use Equation (8-2) to determine the monthly breakeven quantity of 7500 gallons.

$$Q_{BE} = \frac{FC}{r - v} = \frac{900}{0.30 - 0.18} = 7500$$

b. At 5000 gallons, the profit is negative at $_300, as determined by Equation (8-1). The revenue curve has a lower slope above this threshold gallonage level. Since Q_{BE} can't be determined directly from Equation (8-2), it is found by equating revenue and total cost relations with the threshold level of 5000 included. If Q_U is termed the breakeven quantity above threshold, the equated R and TC relations are

$$0.30(5000) + 0.20(Q_U) = 900 + 0.18(5000 + Q_U)$$
$$Q_U = \frac{900 + 900 - 1500}{0.20 - 0.18} = 15.000$$

Therefore, the required volume per site is 20,000 gallons per month, the point at which revenue and total cost break even at $4500. Figure (8-4) details the relations and points.

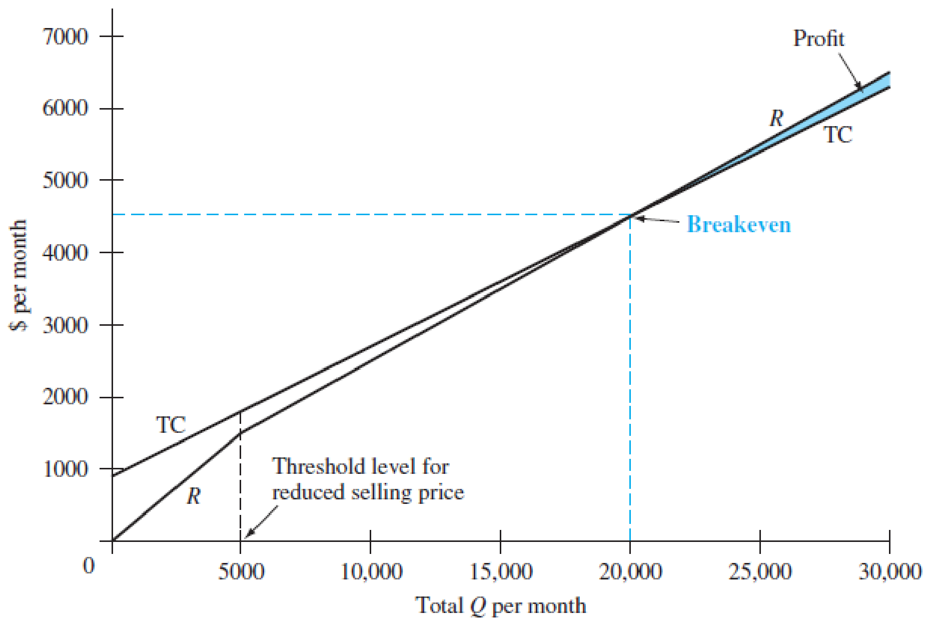


Figure (8-4): Breakeven graph with a volume discount for sales, Example (8-1b).

In some circumstances, breakeven analysis of revenue and cost is better if performed on a per unit basis. The revenue per unit is $R/Q = r$, and the TC relation is divided by Q to obtain cost per unit, also termed average cost per unit C_U.

$$C_U = \frac{TC}{Q} = \frac{FC + vQ}{Q} = \frac{FC}{Q} + v \qquad (8-3)$$

The relation $R/Q = TC/Q$ is solved for Q. The result for Q_{BE} is the same as Equation (8-2).

8.2. Breakeven Analysis Between Two Alternatives.

Breakeven analysis is an excellent technique with which to determine the value of a parameter that is common to two alternatives. The parameter can be the interest rate, capacity per year, first cost, annual operating cost, or any parameter. We have already performed breakeven analysis between alternatives in Chapter 6 for the incremental ROR value (Δi^*).

Breakeven analysis usually involves revenue or cost variables common to both alternatives. Figure (8-5) shows two alternatives with linear total cost (TC) relations. The fixed cost of alternative 2 is greater than that of alternative 1. However, alternative 2 has a smaller variable cost, as indicated by its lower slope. If the number of units of the common variable is greater than the breakeven amount, alternative 2 is selected, since the total cost will be lower. Conversely, an anticipated level of operation below the breakeven point favors alternative 1.

It is common to find the breakeven value by equating PW or AW equivalence relations. The AW is preferred when the variable units are expressed on a yearly

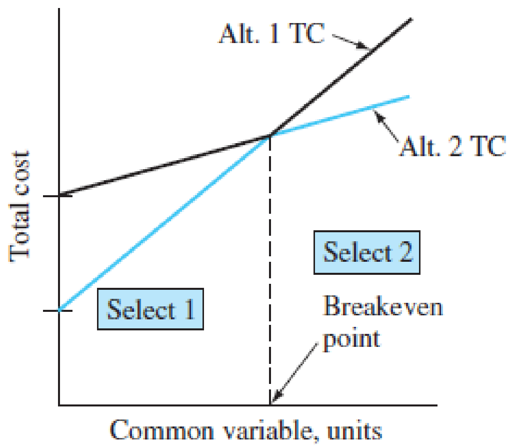


Figure (8-5): Breakeven between two alternatives with linear cost relations.

Example (8-2)

A small aerospace company is evaluating two alternatives: the purchase of an automatic feed machine and a manual feed machine for a finishing process. The auto-feed machine has an initial cost of $23,000, an estimated salvage value of $4000, and a predicted life of 10 years. One person will operate the machine at a rate of $24 per hour. The expected output is 8 tons per hour. Annual maintenance and operating cost is expected to be $3500.

The alternative manual feed machine has a first cost of $8000, no expected salvage value, a 5-year life, and an output of 6 tons per hour. However, three workers will be required at $12 per hour each. The machine will have an annual maintenance and operation cost of $1500. All projects are expected to generate a return of 10% per year. How many tons per year must be finished in order to justify the higher purchase cost of the auto-feed machine?

Solution

Use the steps above to calculate the breakeven point between the two alternatives.
1. Let x represent the number of tons per year.
2. For the auto-feed machine the annual variable cost is

$$Annual\ VC = \frac{\$24}{hour}\frac{1\ hour}{8\ tons}\frac{x\ tons}{year} = 3x$$

The AW expression for the auto-feed machine is

$$AW_{auto} = -23{,}000(A/P, 10\%, 10) + 4000(A/F, 10\%, 10) - 3500 - 3x$$
$$= \$-6992 - 3x$$

Similarly, the annual variable cost and AW for the manual feed machine are

$$Annual\ VC = \frac{\$12}{hour}(3\ operators)\frac{1\ hour}{6\ tons}\frac{x\ tons}{year} = 6x$$
$$AW_{manual} = -8000(A/P, 10\%, 5) - 1500 - 3x$$
$$= \$-3610 - 6x$$

3. Equate the two cost relations and solve for x.

$$AW_{auto} = AW_{manual}$$
$$\$-6992 - 3x = \$-3610 - 6x$$
$$x = 1127\ tons\ per\ year$$

4. If the output is expected to exceed 1127 tons per year, purchase the auto-feed machine, since its VC slope of 3 is smaller.

Problems

1- The fixed costs at Harley Motors are $1 million annually. The main product has revenue of $9.90 per unit and $4.50 variable cost. Determine the following:

a. Breakeven quantity per year
b. Annual profit if 150,000 units are sold
c. Annual profit if 480,000 units are sold

2- A professional photographer who specializes in wedding-related activities paid $16,000 for equipment that has a $2000 estimated salvage value after five years. He estimates that his costs associated with each event amount to $65 per day. If he charges $300 per day for his services, how many days per year must he be employed in order to break even at an interest rate of 8% per year?

3- An independent over-the-road (OTR) truck driver owner paid $98,000 for a used tractor-trailer. The salvage value of the rig after five more years of use is expected to be $66,000. The operating cost is $0.60 per mile and the base mileage rate (revenue) is $0.71 per mile.

a. How many miles per year must the owner drive just to break even at an interest rate of 10% per year?
b. If the owner drives 550 miles per day, how many days per year will be required for breakeven?

4- A semiautomatic process has fixed costs of $40,000 per year and variable costs of $30 per unit. An automatic process has fixed costs of $88,000 per year and variable costs of $22 per unit. At what production level per year will the two alternatives break even?

5- Rent-A-Wreck car rental agency has a contract with PM Warranty, Inc. to do major repairs for $700 per car. The car rental agency estimates that it could repair its own cars for $300 each if it acquires a facility for $300,000 now. A salvage value of $40,000 after 15 years is estimated for the facility. What is the minimum number of cars that must be repaired each year to make the acquisition attractive at an interest rate of 10% per year?

6- Arural 2-lane road can be surfaced with concrete for $2.3 million per mile. If signing, mowing, and winter maintenance are not included, the basic maintenance costs for concrete and asphalt roadways are $483 and $774 per mile per year, respectively. If concrete lasts 20 years, what is the maximum amount that should be spent on asphalt that will last only 10 years? Use an interest rate of 8% per year.

Chapter 9

Depreciation Methods

Chapter 9
Depreciation Methods

General Objective:

Trainee will be able to understand the specific method to reduce the book value of capital invested in an asset or natural resource.

Estimated time for this chapter: 3 Training hours

Detailed Objectives:

1. Depreciation Terminology.
2. Straight Line (SL) Depreciation.
3. Declining Balance (DB).
4. Using Spreadsheets for Depreciation Computation.

9.1. Depreciation Terminology

Primary terms used in depreciation are defined here.

Depreciation is the reduction in value of an asset. The method used to depreciate an asset is a way to account for the decreasing value of the asset to the owner *and* to represent the diminishing value (amount) of the capital funds invested in it. The annual depreciation amount *Dt* does not represent an actual cash flow, nor does it necessarily reflect the actual usage pattern.

Book depreciation and **tax depreciation** are terms used to describe the purpose for reducing asset value. Depreciation may be performed for two reasons:

1. Use by a corporation or business for internal financial accounting.
   This is book depreciation.
2. Use in tax calculations per government regulations. This is tax depreciation.

The methods applied for these two purposes may or may not be the same. *Book depreciation* uses methods to indicate the reduced investment in an asset throughout its expected useful life. The amount of *tax depreciation* is important in an after-tax engineering economy study because the annual tax depreciation is usually tax deductible; that is, it is subtracted from income when calculating the amount of income taxes.

Tax depreciation may be calculated and referred to differently in countries outside the United States. For example, in Canada the equivalent is CCA (capital cost allowance), which is calculated based on the undepreciated value of all corporate properties that form a particular class of assets, whereas in the United States, depreciation may be determined for each asset separately.

- **First cost *P*** or **basis *B*** is the delivered and installed cost of the asset including purchase price, installation fees, and any other depreciable direct costs.
- **Book value BV** represents the remaining, undepreciated investment after the total amount of depreciation charges to date have been removed. The book value is determined at the end of each year, which is consistent with the end-of-year convention.
- **Recovery period *n*** is the depreciable life in years. Often there are different *n* values for book and tax depreciation. Both of these values may be different from the asset's estimated productive life.
- **Market value MV** is the estimated amount realizable if the asset were sold on the open market. Because of the structure of depreciation laws, the book value and market value may be substantially different. For example, a commercial building tends to increase in market value, but the book value will decrease with time. However, IT equipment usually has a market value much lower than its book value due to rapidly changing technology.
- **Salvage value *S*** is the estimated trade-in or market value at the end of the asset's depreciable life. The salvage value, expressed as a dollar amount or a percentage of first cost, may be positive, zero, or negative (due to carry-away costs).
- **Depreciation rate** or **recovery rate *dt*** is the fraction of the first cost removed by depreciation each year.
- **Personal property,** one of the two types of property for which depreciation is allowed, is the income-producing, tangible possessions of a corporation. Examples are vehicles, manufacturing equipment, computer equipment, chemical processing equipment, and construction assets.

- **Real property** includes real estate and all improvements office buildings, factories, warehouses, apartments, and other structures. *Land itself is considered real property, but it is not depreciable.*
- **Half-year convention** assumes that assets are placed in service or disposed of in midyear, regardless of when these events actually occur. This convention is utilized in U.S. required tax depreciation methods.

9.2. Straight Line (SL) Depreciation.

Straight line is considered the standard against which any depreciation method is compared. It derives its name from the fact that the book value decreases linearly with time. For book depreciation purposes, it offers an excellent representation of book value for any asset that is used regularly over a number of years.

The annual SL depreciation is determined by multiplying the first cost minus the salvage value by the depreciation rate.

$$D_t = (B - S)d = \frac{(B - S)}{n} \qquad (9-1)$$

Where D_t depreciation charge for year t $(t = 1, 2, \ldots, n)$
B = basis or first cost
S = estimated salvage value
n = recovery period
d = depreciation rate = $1/n$

Since the asset is depreciated by the same amount each year, the book value after t years of service, denoted by BV_t, is the basis B minus the annual depreciation times t.

$$BV_t = (B - tD_t) \qquad (9-2)$$

The depreciation rate for a specific year is d_t. However, the SL model has the same rate for all years.

$$d = d_t = \frac{1}{n} \qquad (9-3)$$

The format for the spreadsheet function to display annual SL depreciation is

$$= SLN(B, S, n)$$

Example (9-1)

If an asset has a first cost of $50,000 with a $10,000 estimated salvage value after 5 years, calculate the annual SL depreciation and plot the yearly book value.

Solution

The depreciation each year for 5 years is

$$D_t = (B - S)d = \frac{(50{,}000 - 10{,}000)}{5} = \$8000$$

The book values, computed using Equation (9-2), are plotted in Figure (9-1).
For year 5, for example,

$$BV_5 = (50{,}000 - 5(8000)) = \$10{,}000 = S$$

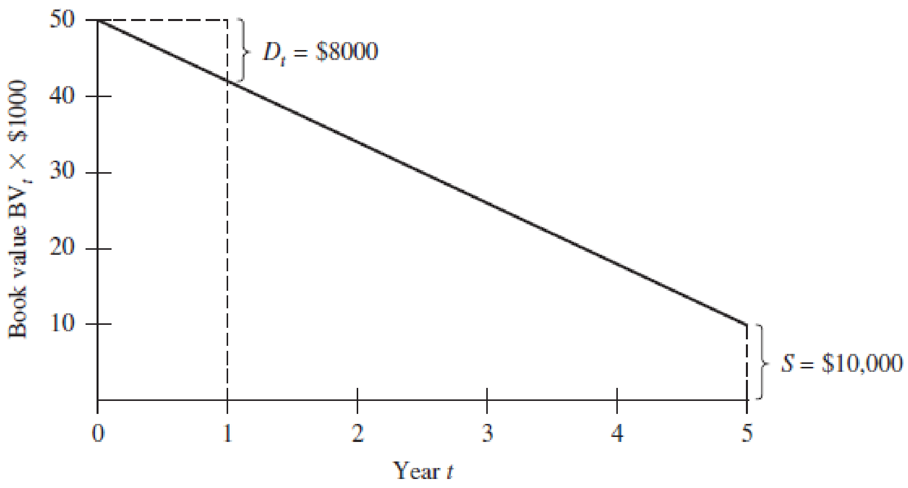


Figure (9-1): Book value of an asset depreciated using the straight line method, Example (9-1).

9.3. Declining Balance (DB).

Declining balance is also known as the fixed percentage or uniform percentage method. DB depreciation accelerates the write-off of asset value because the annual depreciation is determined by multiplying the book value at the beginning of a year by a fixed percentage d, expressed in decimal form. If $d = 0.2$, then 20% of the book value is removed each year. The amount of depreciation decreases each year.

The maximum annual depreciation rate for the DB method is twice the straight line rate.

$$d_{max} = \frac{2}{n} \qquad (9-4)$$

This is called double declining balance (DDB). If $n = 5$ years, the DDB rate is 0.4; so 40% of the book value is removed annually. Another commonly used percentage for the DB method is 150% of the SL rate, where $d = 1.5/n$. The depreciation for year t is the fixed rate d times the book value at the end of the previous year.

$$D_t = (d)\, BV_{t-1} \qquad (9-5)$$

Book value in year t is determined by

$$BV_t = B(1-d)^t \qquad (9-6)$$

The actual depreciation rate for each year t, relative to the first cost, is

$$d_t = B(1-d)^{t-1} \qquad (9-7)$$

If BV_{t-1} is not known, the depreciation in year t can be calculated using B and d_t from Equation (9-7).

$$D_t = dB(1-d)^{t-1} \qquad (9-8)$$

It is important to understand that the DB book value never goes to zero, because the book value is always decreased by a fixed percentage. The implied salvage value after *n* years is the BV_n amount.

$$Implied\ S = BV_n = B(1-d)^n \qquad (9-9)$$

If a salvage value is initially estimated, this value is not used in the DB or DDB method. However, if the implied S < estimated S, it is correct to stop charging further depreciation when the book value is at or below the estimated salvage value.

The spreadsheet functions DDB and DB display depreciation amounts for specific years. The formats are

$$= DDB(B, S, n, t, d)$$

$$= DB(B, S, n, t)$$

The d is a number between 1 and 2. If omitted, it is assumed to be 2 for DDB. The DDB function automatically checks to determine when the book value equals the estimated S value. No further depreciation is charged when this occurs.

Example (9-2)

Albertus Natural Stone Quarry purchased a computer-controlled face-cutter saw for $80,000. The unit has an anticipated life of 5 years and a salvage value of $10,000.

a. Compare the schedules for annual depreciation and book value using two methods: DB at 150% of the straight-line rate and at the DDB rate.
b. How is the estimated $10,000 salvage value used?

Solution

a. The DB depreciation rate is $d = 1.5/5 = 0.30$ while the DDB rate is $d_{max} = 2/5 = 0.40$. Table (9-1) and Figure (9-2) present the comparison of depreciation and book value. Example calculations of depreciation and book value for each method follow.

Table (9-1): Annual Depreciation and Book Value, Example (9-2)

	Declining Balance, d = 0.30		Double Declining Balance, *d = 0.40*	
Year,t	**D_t**	**BV_t**	**D_t**	**BV_t**
0		$80,000		$80,000
1	$24,000	56,000	$32,000	48,000
2	16,800	39,200	19,200	28,800
3	11,760	27,440	11,520	17,280
4	$8,232 8	19,208	6,912	10,368
5	5,762	13,446	368	10,000

150% DB for year 2 by Equation (9-5) with $d = 0.30$

$$D_2 = 0.30(56{,}000) = \$16{,}800$$

$$BV_2 = 80{,}000(0.70)^2 = \$39{,}200$$

DDB for year 3 by Equation (9-5) with $d = 0.40$

$$D_3 = 0.40(28{,}000) = \$11{,}520$$

$$BV_3 = 80{,}000(0.60)^2 = \$17{,}280$$

The DDB depreciation is considerably larger during the first years, causing the book values to decrease faster, as indicated in Figure (9-2).

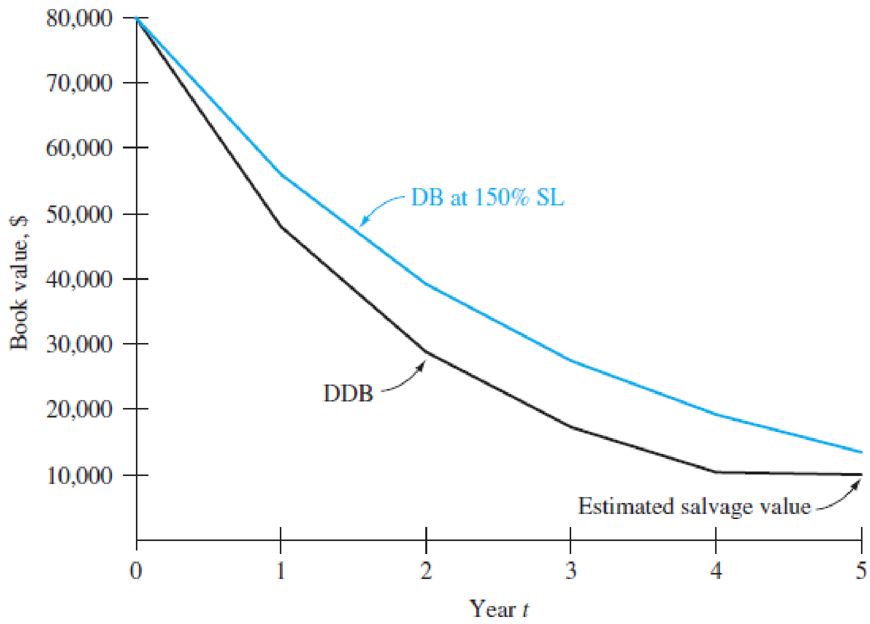


Figure (9-2): Plot of book values for two declining balance methods, Example (9-2).

9.4. Using Spreadsheets for Depreciation Computation.

Spreadsheet functions calculate the annual depreciation for methods discussed and for switching between declining balance and straight line. The next example illustrates the functions in the same order in which the methods were discussed. The final part illustrates the use of an x-y scatter chart of book values to compare methods.

Example (9-3)

BA Aerospace purchased new aircraft engine diagnostics equipment for its maintenance support facility in France. Installed cost was $500,000 with a depreciable life of 5 years and an estimated 1% salvage. Use a spreadsheet to determine the following:

a. Straight line (SL) depreciation and book value schedule.
b. Double declining balance (DDB) depreciation and book value schedule.
c. Declining balance (DB) at 150% of the SL rate depreciation and book value schedule.

Solution

Assume that any method can be used to depreciate this equipment. All five schedules and the graph may be developed using a single worksheet; however, several are developed here to show the progression through different methods. The functions for depreciation applied here SLN, DDB, and VDB.

a. Figure (9-3): The SLN function, which determines SL depreciation in column B, has the same entry each year. The entire amount $B - S = \$495{,}000$ is depreciated. Yearly book value for this and all other methods is determined by subtraction of annual depreciation D_t from BV_{t-1}.
b. Figure (9-4): Column B details double declining balance depreciation using the $DDB\ function = DDB(500000{,}5000{,}5, t, 2)$ in which the period value t changes each year. The last entry is optional; if omitted, 2 for DDB is assumed. $BV_5 = \$38{,}880$ does not reach the estimated salvage of $5000.
c. Figure (9-4): Declining balance depreciation at 150% of the SL rate (column D) is best determined using the DDB function with the last field entered as 1.5. The ending book

value (column E) is now higher at \$84,035 since the annual depreciation rate is $1.5/5 = 0.3$, compared to the DDB rate of 0.4.

	A	B	C	D	E	F
1	Basis	\$500,000		Recovery	5 years	
2	Salvage	\$5000		Method	SL	
3						
4		Straight line				
5	Year, t	D_t, \$	BV_t, \$			
6	0		500,000			
7	1	99,000	401,000			
8	2	99,000	302,000			
9	3	99,000	203,000			
10	4	99,000	104,000			
11	5	99,000	5000			
12	Total	495,000				
13						
14		Straight line method				
15						

Book value = C6-B7

Straight line function = SLN(500000,5000,5)

Figure (9-3): Straight line depreciation and book value schedule, Example (9-3a).

	A	B	C	D	E
1	First cost	\$500,000	Recovery	5 years	
2	Salvage	\$5000	Methods	DDB and DB at 150% SL	
3					
4		Double declining balance		Declining balance at 150% SL	
5	Year, t	D_t, \$	BV_t, \$	D_t, \$	BV_t, \$
6	0		500,000		500,000
7	1	200,000	300,000	150,000	350,000
8	2	120,000	180,000	105,000	245,000
9	3	72,000	108,000	73,500	171,500
10	4	43,200	64,800	51,450	120,050
11	5	25,920	38,880	36,015	84,035
12	Total	461,120		415,965	
13					
14					
15					
16					
17					
18		Declining balance methods			
19					

DDB function, year 5 = DDB(500000,5000,5,5,2)

DB function for 150% SL, year 5 = DDB(500000,5000,5,5,1.5)

Figure (9-4): DDB and DB at 150% SL rate depreciation and book value schedules, Example (9-3b and c).

Problems

1- How does depreciation affect income taxes?
2- What are three depreciable costs that are included in an asset's basis?
3- What is the difference between book value and market value?
4- There are 3 different life values (recovery periods) associated with a depreciable asset. Identify each by name and explain how it is correctly used.
5- A company that manufactures pulse Doppler insertion flow meters uses the straight-line method for book depreciation purposes. Newly acquired equipment has a first cost of $170,000 with a 3-year life and $20,000 salvage value. Determine the depreciation charge and book value for year two.
6- Butler Buildings purchased semi-automated assembly and riveting robotics equipment for constructing its modular warehouse buildings. The first cost was $475,000 and installation costs were $75,000; life is estimated at 10 years with a salvage of 15% of first cost. Use the SL method to determine
   a. annual recovery rate,
   b. annual depreciation,
   c. book value after 5 years,
   d. book value after 10 years.
7- Halcrow Yolles purchased equipment for new highway construction in Manitoba, Canada, costing $500,000 Canadian. Estimated salvage at the end of the expected life of 5 years is $50,000. Various book depreciation methods are being studied currently. Determine the depreciation for year 2 using the DDB, 150% DB and SL methods.
8- An engineer with Accenture Middle East BV in Dubai was asked by her client to help understand the difference between 150% DB and DDB depreciation. Answer the questions if B= $180,000, n = 12 years, and S = $30,000.
   a. What are the book values after 12 years for both methods?
   b. How do the estimated salvage and the two book values after 12 years compare in value?
   c. Which of the two methods, when calculated correctly considering S = $30,000, writes off more of the first cost over the 12 years?

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